

Handout 2

Introduction to Communication Systems

Solutions to Homework 1

September 24, 2009

PROBLEM 1. 1. $H(R) = 2$, $H(P) = 1.8016$ and $H(Q) = 1.9406$.

2. We know that the discrete entropy is maximized by the uniform distribution. Therefore, $H(R)$ has the maximum entropy. By looking at P and Q we can see that Q is closer to the uniform distribution than P . We can then conclude that $H(R) > H(Q) > H(P)$.

PROBLEM 2. 1. $\sum_{i=1}^{\infty} p_i = \alpha \sum_{i=1}^{\infty} \frac{1}{2^{i+1}} = \frac{\alpha}{2} (\sum_{i=0}^{\infty} \frac{1}{2^i} - 1) = \frac{\alpha}{2} (\frac{1}{1-1/2} - 1) = \frac{\alpha}{2} = 1$. This means that we need $\alpha = 2$.

2. $\sum_{i=1}^{\infty} p_i \log_2(\frac{1}{p_i}) = \sum_{i=1}^{\infty} \frac{1}{2^i} \log_2(2^i) = \sum_{i=1}^{\infty} \frac{1}{2^i} i$
 $= \sum_{i=0}^{\infty} \frac{1}{2^i} i = \frac{1/2}{(1-1/2)^2} = 2$.