ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 12 Advanced Digital Communications
Homework 6 November 6, 2009

PROBLEM 1.

$$Q(D) = (1 - e^{-4})\frac{1}{1 - e^{-2}D}\frac{1}{1 - e^{-2}D^{-1}}$$

and whitening filter is

$$W(D) = (1 - e^{-4})\frac{1}{1 - e^{-2}D}\frac{1}{1 - e^{-2}D^{-1}}$$

PROBLEM 2.

$$Q(D) = (1 - \frac{1}{2}D)(1 - \frac{1}{2}D^{-1})$$

and

$$F(D) = 1 - \frac{1}{2}D$$

So

$$y_k = \frac{||p||}{\sqrt{N_0}}(x_k - \frac{1}{2}x_{k-1}) + w_k$$

PROBLEM 3. We know

$$S_z(D) = S_u(D)C(D)C^*(D^{-*})$$

also we can compute

$$S_z(D) = (1 - 0.9D)(1 - 0.9D^{-1})$$

so

$$C(D) = (1 - 0.9D)$$

is the coloring filter.

PROBLEM 4. Consider transmission over an ISI channel with PAM and symbol period T. Let $\phi(t) = \frac{1}{\sqrt{T}}\text{sinc}(\frac{t}{T})$ and $h(t) = \delta(t) - \frac{1}{2}\delta(t - T)$. Assume that AWGN noise has power spectral density N_0.

(a)

$$p(t) = \frac{1}{\sqrt{T}}\text{sinc}(\frac{t}{T}) - \frac{1}{2\sqrt{T}}\text{sinc}(\frac{t - T}{T})$$

(b)

$$||p|| = \int_{-\infty}^{\infty} p^2(t)\, dt = \frac{5}{4}$$

(c)

$$E(Z_n Z_{n-k}^*) = E(\int z(t)\hat{\phi}_n(t)\, dt \int z(t)\hat{\phi}_{n-k}(t)\, dt) = N_0 q_k$$

where $q_0 = 1$ and $q_1 = q_{-1} = -\frac{2}{5}$. So the whitening filter is

$$G(D) = \sqrt{\frac{5}{4}}\frac{1}{1 - \frac{1}{2}D^{-1}}$$

so the resulting channel is

$$y_k = x_k - \frac{1}{2}x_{k-1} + w_k$$

(d)