Handout 9 Advanced Digital Communications
Homework 3 October 19, 2009

PROBLEM 1.

(a) $[1,1,-1,1],[1,1,-1,-1],[-1,-1,-1,-1],[-1,-1,1,1],$
$[1,-1,1,-1],[1,-1,-1,1],[-1,1,1,-1],[-1,1,-1,1]$

(b)

$$b = \log 8 = 3 \tag{1}$$

$$\bar{b} = \frac{\log 8}{4} = \frac{3}{4} \tag{2}$$

(c)

$$E_x = 4 \tag{3}$$

$$\overline{E}_x = 1 \tag{4}$$

(d) Each point has 6 neighbors with distance $d_{min}^2 = 8$

$$P_e \leq 6 * Q(\frac{d_{min}}{2\sigma}) = 6Q(\frac{2\sqrt{2}}{2\sqrt{0.1}}) \tag{5}$$

$$= 6Q(\sqrt{\frac{2}{0.1}}) = 6Q(\sqrt{20}) \tag{6}$$

(which is quite small)

PROBLEM 2.

1.

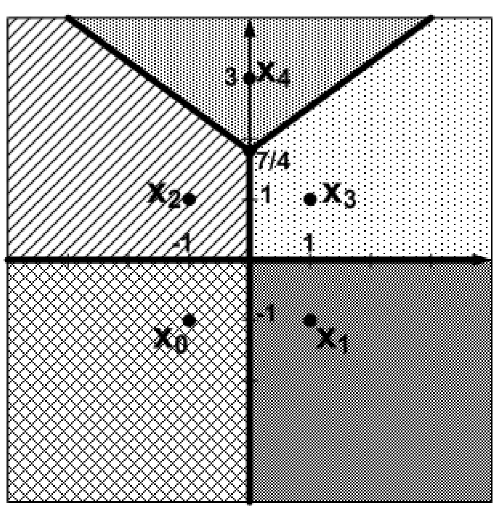


Figure 1: Decision Regions for the ML detector

2. Union bound:

$$P_e < \frac{1}{5}(P(error|x_4) + P(error|x_3) + P(error|x_2) + P(error|x_1) + P(error|x_0)) \quad (7)$$
$$< (M-1)Q(\frac{d_{min}}{2\sigma}) = 4Q(\frac{1}{\sigma}) \quad (8)$$

Nearest neighbor union bound:

$$P_e < M_{d_{min}} Q(\frac{d_{min}}{2\sigma}) = 2Q(\frac{1}{\sigma}) \quad (9)$$

PROBLEM 3.

1.

$$x(t) = a\cos(2\pi(f_c + \frac{1}{T})t) + b\cos(2\pi(f_c + \frac{2}{T})t) \quad (10)$$
$$= (a\cos(\frac{2\pi}{T}t) + b\cos(\frac{4\pi}{T}t))\cos(2\pi f_c t) \quad (11)$$
$$-(a\sin(\frac{2\pi}{T}t) + b\sin(\frac{4\pi}{T}t))\sin(2\pi f_c t) \quad (12)$$
$$= x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t) \quad (13)$$

Therefore, we have

$$x_{bb}(t) = x_I(t) + jx_Q(t) \quad (14)$$
$$= a\cos(\frac{2\pi}{T}t) + ja\sin(\frac{2\pi}{T}t) + b\cos(\frac{4\pi}{T}t) + jb\sin(\frac{4\pi}{T}t) \quad (15)$$
$$= a\exp j\frac{2\pi}{T}t + b\exp j\frac{4\pi}{T}t \quad (16)$$

Where $\{\exp j\frac{2\pi}{T}t, \exp j\frac{4\pi}{T}t\}$ form an orthonormal basis.

2.

$$\overline{x}_{bb} = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\varphi_1(t) = \exp j\frac{2\pi}{T}t \quad (17)$$
$$\varphi_2(t) = \exp j\frac{4\pi}{T}t \quad (18)$$

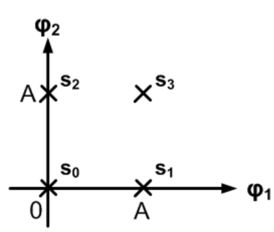


Figure 2: Signal Constellation for $\overline{x}_{bb}$

3.

$$E_{bb} = \frac{1}{4}\sum_{i=0}^{3} E_{s_i} = \frac{1}{4}\sum_{i=0}^{3} ||s_i||^2 \quad (19)$$

$$= \frac{1}{4}(0 + A^2 + A^2 + 2A^2) = A^2 \quad (20)$$

No, this is not a minimum energy constellation. We get the minimum energy constellation by shifting the signal set by $-\frac{1}{4}\sum_{i=0}^{3}\vec{s_i} = -\frac{1}{2}\begin{pmatrix} A \\ A \end{pmatrix}$; and the origin will be the center of this constellation.

PROBLEM 4.

a)

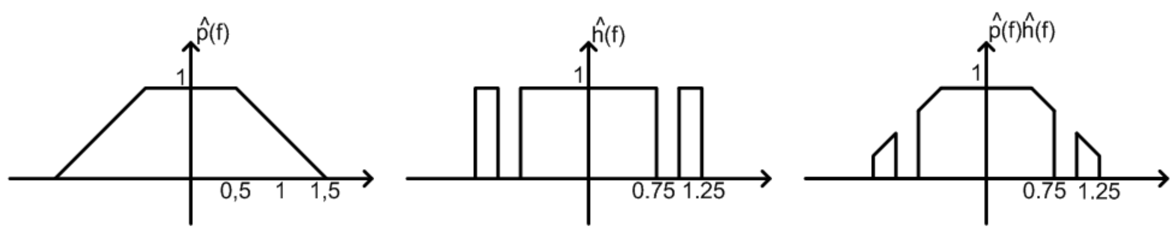
$$g(kT) = \begin{cases} 1 & , \quad k = 0 \\ 0 & , \quad k \neq 0 \end{cases}$$

Nyquist criterion:
If $\hat{g}(f)$ satisfies

$$\sum_{k=-\infty}^{\infty} |\hat{g}(f - \frac{k}{T})| = T$$

we'll get a perfect reconstruction.

b)



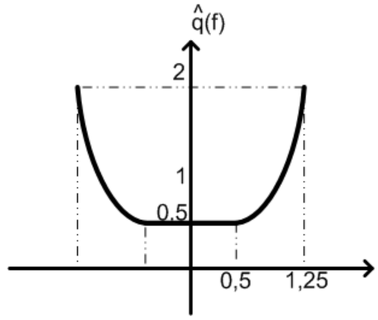
Yes, it is possible to find $\hat{q}(f)$'s for which there is no intersymbol interference.
When

$$\hat{q}(f) = \begin{cases} \frac{1}{2} & , \quad |f| \leq 0.5 \\ \frac{1}{2(1.5-f)} & , \quad 0.5 < |f| \leq 0.75 \quad \& \quad 1 < |f| \leq 1.25 \end{cases}$$

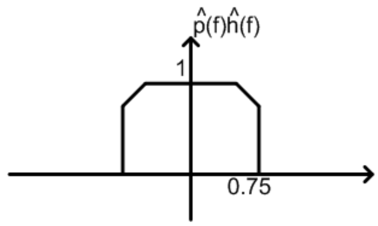
$\hat{g}(f)$ satisfies the Nyquist criterion, i.e.

$$\sum_{k=-\infty}^{\infty} |\hat{g}(f - 2k)| = T = \frac{1}{2}$$

and the solution for $\hat{q}(f)$ is non-unique for the intervals $0.75 < |f| \leq 1$ and $|f| > 1.25$
A possible $\hat{q}(f)$ which satisfies the above criteria is as below:



c) This time we have



and there is no possible solution for $\hat{q}(f)$ so that $\hat{g}(f)$ satisfies the Nyquist criterion; i.e. $\sum_{k=-\infty}^{\infty} |\hat{g}(f-2k)| = \frac{1}{2}$ cannot be achieved for any $\hat{q}(f)$ chosen.

d) Intersymbol interference can be avoided by proper choice of $\hat{q}(f)$ iff

$$\sum_{k=-\infty}^{\infty} \hat{p}(f-2k)\hat{h}(f-2k) \neq 0 \quad, \forall f \tag{21}$$