ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 9 Advanced Digital Communications
Homework 6 October 26, 2009

Problem 1. Let the transmission over an ISI channel yield after matched filtering the following model,

$$Y(D) = ||p||X(D)Q(D) + Z(D)$$

where $q_l = e^{-|2l|}$ and $S_z(D) = N_0 Q(D)$ and $Q(D)$ is the D-Transform of $\{q_l\}$. Find the whitening filter $W(D)$ to whiten the noise. Choose the whitening filter such that the resulting communication channel after the whitening filter is causal. That is, $Q(D)W(D)$ is causal.

Problem 2. Suppose we are given q_k, (the autocorrelation function of the normalized pulse function) by : $q_0 = \frac{5}{4}$, $q_1 = q_{-1} = \frac{1}{2}$, and where the equivalent channel in D-transform resulting out of matching filter is given by :

$$Y(D) = \frac{1}{N_0} S_z(D)||p||X(D) + Z(D)$$

Find $S_z(D)$, $F(D)$ and the resulting channel (write in the temporal domain $G(D)Y(D)$).

Problem 3. Let $\{Z_n\}$ be a wide-sense stationary random process with $EZ_n = 0$,

$$EZ_nZ_{n-l} = \begin{cases} 1.181 & l = 0; \\ 0.9 & |l| = 1; \\ 0 & \text{other wise} \end{cases}$$

and let $\{Z_n\}$ be a real process. Now let $\{U_n\}$ be a white, wide-sense stationary real random process, i.e $EU_n = 0$, and $EU_nU_{n-l} = \begin{cases} 1 & l = 0; \\ 0 & \text{other wise} \end{cases}$ Find a coloring filter $\{C_n\}$ such that $Z_n = C_n * U_n$

Problem 4. Consider transmission over an ISI channel with PAM and symbol period T. Let $\phi(t) = \frac{1}{\sqrt{T}}\text{sinc}(\frac{t}{T})$ and $h(t) = \delta(t) - \frac{1}{2}\delta(t - T)$. Assume that AWGN noise has power spectral density N_0.

(a) Determine the pulse response p(t).

(b) Determine $||p||$ and $\hat{\phi}(t)$.

(c) Find the autocorrelation function of the noise after sampling the output of the matched filter. Find the whitening filter such that the resulting channel is causal.

(d) Assume that $N_0 = \frac{25}{64}$, size of PAM is 2 and $x_i \in \{-1; 1\}$. Let the transmitted sequence be $\{1; -1; -1; 1; 1\}$ and the output of the whitened matched filter is $\{0.7; 0.1; -2.0; 0.4; 0.7\}$. Find the maximum likelihood sequence using the Viterbi algorithm. Assume that the initial and last states are 1.

Hint. The set of discrete-time samples $\{y(kT)\}_{k\in(-\infty,\infty)}$ where $y(kT) = \hat{y}(t) * \hat{\phi}^*(-t)|_{t=kT}$ is a set of sufficient statistics for the detection of $\{x_k\}$ over a continuous-time AGN ISI channel whose output is $\hat{y}(t)$, i.e.,

$$\hat{y}(t) = h(t) * \sum_n x_n \phi(t - nT) + z(t),$$

$p(t) = h(t) * \phi(t)$ and $\hat{\phi}(t) = \frac{h(t)*\phi(t)}{||h*\phi||}$.