ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
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Handout 5 Advanced Digital Communications
Homework 3 October 2, 2009

Problem 1. A set of 4 orthogonal basis functions $\phi_1(t); \phi_2(t); \phi_3(t); \phi_4(t)$ is used in the following constellation. In both the first 2 dimensions and again in the second two dimensions: The constellation points are restricted such that a point E can only follow a point E and a point O can only follow a point O. The points $\{1, 1\}$, $\{-1, -1\}$ are labeled as E and $\{1, -1\}$, $\{-1, 1\}$ are labeled as O points. For instance, the 4-dimensional point $[1, 1, 1, 1]$ is permitted to occur, but the point $[1, 1, -1, 1]$ can not occur.

1. Enumerate all M points as ordered-4-tuples.
2. Find b, $\bar{b}$ (number of transmitted bits and number of transmitted bits per dimension).
3. Find E_x and $\bar{E}_x$ (energy and energy per dimension) for this constellation.
4. Find P_e for this constellation using the nearest neighbor union bound if used on an AWGN with $\sigma^2 = 0.1$.

Problem 2. Consider the following constellation to be used on an AWGN channel with variance σ^2:

$$\begin{aligned} x_0 &= (-1, -1) \\ x_1 &= (1, -1) \\ x_2 &= (-1, 1) \\ x_3 &= (1, 1) \\ x_4 &= (0, 3) \end{aligned}$$

1. Find the decision region for the ML detector.
2. Find the union bound and nearest neighbor union bound on P_e for the ML detector on this signal constellation.

Problem 3. Let the transmitted bandpass signal be given by

$$x(t) = a\cos\big(2\pi(f_c + \frac{1}{T})t\big) + b\cos\big(2\pi(fc + \frac{2}{T})t\big), \qquad t \in [0, T]$$

and $a \in \{0, A\}$, $b \in \{0, A\}$.

1. Find the baseband equivalent signal $x_{bb}(t)$ for the transmitted signal.
2. Find the vector representation of the baseband signal and draw the corresponding signal constellation.
3. If $a = \begin{cases} 0 & \text{w.p.}\frac{1}{2} \\ A & \text{w.p.}\frac{1}{2} \end{cases}$ and $b = \begin{cases} 0 & \text{w.p.}\frac{1}{2} \\ A & \text{w.p.}\frac{1}{2} \end{cases}$.

   Find the average energy of the baseband signal. Is this a minimum energy configuration? If not how will you modify the constellation so that it is of minimum energy?

PROBLEM 4. Consider a PAM baseband system in which the modulator is defined by a signal interval T and a waveform $p(t)$, the channel is defined by a filter $h(t)$, and the receiver is defined by a filter $q(t)$ which is sampled at T-spaced intervals. The received waveform, after the receiver filter $q(t)$, is then given by $r(t) = \sum_k u_k g(t - kT)$, where $g(t) = p(t) * h(t) * q(t)$.

(a) What property must $g(t)$ have so that $r(kT) = u_k$ for all k and for all choices of input $\{u_k\}$? What is the Nyquist criterion for $\hat{g}(f)$?

(b) Now assume that $T = \frac{1}{2}$ and that $p(t), h(t), q(t)$ and all their Fourier transforms are restricted to be real. Assume further that $\hat{p}(f)$ and $\hat{h}(f)$ are specified by

$$\hat{p}(f) = \begin{cases} 1 & |f| \leq 0.5 \\ 1.5 - |f| & 0.5 \leq |f| \leq 1.5 \\ 0 & |f| > 0 \end{cases}$$

and

$$\hat{h}(f) = \begin{cases} 1 & |f| \leq 0.75 \\ 0 & 0.75 \leq |f| \leq 1 \\ 1 & 1 \leq |f| \leq 1.25 \\ 0 & |f| > 0 \end{cases}.$$

Is it possible to choose a receiver filter transform $\hat{q}(f)$ so that there is no intersymbol interference? If so, give such a $\hat{q}(f)$ and indicate the regions in which your solution is nonunique.

(c) Redo part (b) with the modification that now $\hat{h}(f) = 1$ for $|f| \leq 0.75$ and $\hat{h}(f) = 0$ for $|f| > 0.75$.

(d) Explain the conditions on $\hat{p}(f)\,\hat{h}(f)$ under which intersymbol interference can be avoided by proper choice of $\hat{q}(f)$. (You may assume, as above, that $\hat{p}(f)$, $\hat{h}(f)$, $p(t)$ and $h(t)$ are all real.)