Homework Set # 3
Principles of Wireless Networks

Problem 1 (Multi Source Network)

Let $\mathcal{N}$ be a network represented by an acyclic directed graph $\mathcal{G} = (V, E)$, where V is the set of nodes and $E = \subseteq \{(i, j) : i, j \in V\}$ is the set of edges in $\mathcal{G}$ and each edge represents a communication channel. Each edge $(i, j) \in E$ is assigned with a non-negative number c_{ij} which denoted its capacity.

Let $\mathcal{S} \subset V$ be the set of source nodes, each wants to transmit its message W_S, $S \in \mathcal{S}$ to the destination node $D \in V$. Let R_S be the rate of transmission from source node S to D, and denote the set of all admissible rate tuples by $\mathcal{R} = \{(R_S, S \in \mathcal{S}) : (R_S, S \in \mathcal{S}) \text{ is admissible}\}$.

(a). Show that any admissible rate tuple $(R_S, S \in \mathcal{S})$ satisfies

$$\sum_{T \in \mathcal{T}} R_T \le \min_{\Omega \in \Lambda(\mathcal{T}, D)} C(\Omega), \qquad \forall \mathcal{T} \subseteq \mathcal{S} \tag{1}$$

where $\Lambda(\mathcal{T}, D) = \{\Omega \subset V : \mathcal{T} \subseteq \Omega, D \in \Omega^c\}$ is the set of cuts which separate the source node set $\mathcal{T}$ from the destination node, and $C(\Omega)$ is the cut-value defined in the lecture as

$$C(\Omega) = \sum_{\substack{(i, j) \in E \\ i \in \Omega,\, j \in \Omega^c}} c_{ij}. \tag{2}$$

(b). Prove that any rate tuple satisfies (1), is achievable, *i.e*, show that there exist communication schemes which guarantees transmission rate R_S for all $S \in \mathcal{S}$ to D, with vanishing error probability. Then conclude the rate region of this network is in fact characterized by (1).
Hint: Use random mapping at the relay nodes and show that the error probability goes to zero, as the field size grows.

Problem 2 (Deterministic Relay Network in Half-Duplex Regime)

Consider the relay network shown in Fig. 1 with $n_{SR} = 3$, $n_{SD} = 2$, and $n_{RD} = 3$. We have seen in the lecture that if the relay node can listen to the source (receive bits from S) and talk to the destination (transmit bits to D), then there exist schemes which guarantees that the destination gets 3 bits per second.

(a). Find the value of the cuts separating source and destination nodes. What is the min-cut of this network?

(b). Consider the full-duplex regime, *i.e.*, the relay node can listen to the source (receive bits from S) and talk to the destination (transmit bits to D), at the same time. What is maximum achievable rate for this regime? Write a scheme which guarantees such rate.

(c). Now, assume that the relay is in half-duplex mode, that is, it cannot listen and talk at the same time, *i.e.*, at each period of time, it can either listen to the sources, or talk to the destination. Show that one can achieve the rate 2.5 bits/second for this regime.

(d). Consider a general network with three nodes and channel gains (n_{SR}, n_{SD}, n_{RD}), wherein the relay listens to the source for α-fraction of time, and talks to the destination for the $(1 - \alpha)$ remaining fraction. Find the maximum achievable rate for this network.

(e). Optimize the result of (b) for α. Compare the maximum transmission rate of the network in the half-duplex regime to that of the full-duplex one.

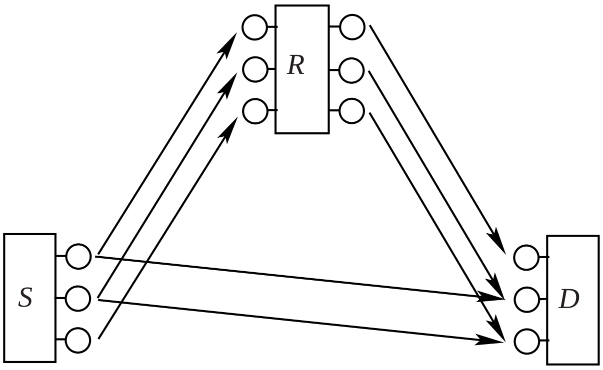

Figure 1: Deterministic relay network: $n_{SR} = 3$, $n_{SD} = 2$, $n_{RD} = 3$.

Problem 3 (Min-Cut value)

Consider the deterministic network shown in Fig. 2.

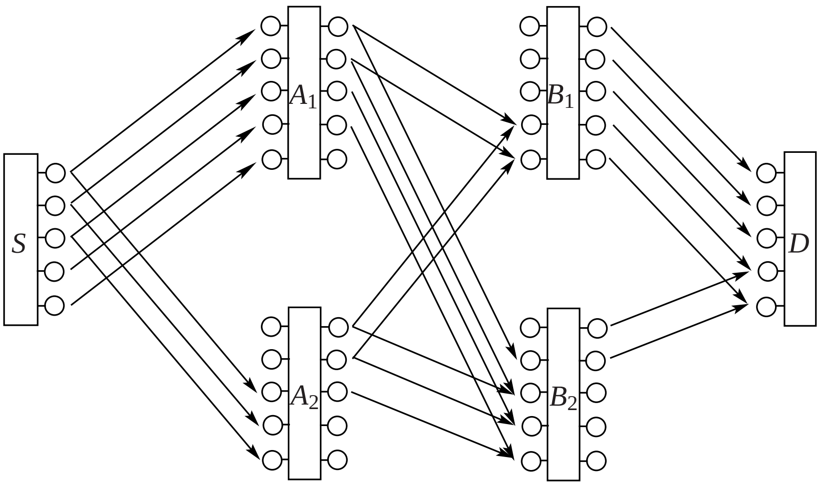

Figure 2: Deterministic network with 6 nodes.

(a). Consider the shift linear deterministic model, wherein the channel between nodes u, and v is $\mathbf{G}_{uv} = \mathbf{S}^{5-n_{uv}}$, where

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (3)$$

and n_{uv} is the channel gain between nodes u, and v, given in Fig. 2. Find the values of all the cuts separating source and destination. What is the min-cut value of this network?

(b). Find the rank of matrix $\mathbf{G}_{A_2B_1}$. Replace this matrix with some other matrix $\mathbf{G}'_{A_2B_1}$ with the same rank (note that $\mathbf{G}'_{A_2B_1}$ does not have to be a power of $\mathbf{S}$), such that min-cut value in the modified network becomes 5.