ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 7 Signal Processing for Communications
Homework 6. March 30, 2009

Problem 1 (Problem 5.6 in the book). Consider the following input-output relations and, for each of the underlying systems, determine whether the system is linear, time invariant, BIBO stable, causal or anti-causal. Characterize the eventual LTI systems by their impulse response.

- $y[n] = x[-n]$.
- $y[n] = e^{-jwn}x[n]$.
- $y[n] = \sum_{k=n-n_0}^{n+n_0} x[k]$.
- $y[n] = ny[n-1] + x[n]$ such that if $x[n] = 0$ for $n < n_0$, then $y[n] = 0$ for $n < n_0$.

Problem 2 (Problem 5.7 in the book). Derive the impulse response of a bandpass filter with center frequency ω_0 and passband ω_b:

$$H_{bp}(e^{jw}) = \begin{cases} 1 & \omega_0 - \omega_b \leq \omega \leq \omega_0 + \omega_b, \\ 1 & -\omega_0 - \omega_b \geq \omega \geq -\omega_0 + \omega_b, \\ 0 & \text{elsewhere.} \end{cases}$$

(Hint: Consider the following ingredients: a cosine of frequency ω_0, a lowpass filter of bandwidth ω_b and the modulation theorem.)

Problem 3. For each of the following sequences $x[n]$, find the z-transform $X(z)$ and the corresponding region of convergence, and sketch the pole-zero diagram.

- $x[n] = \delta[n-3] + \delta[n+3]$
- $x[n] = na^n u[n]$
- $x[n] = -na^n u[-n-1]$
- $x[n] = 2^n u[n] - 3^n u[-n-1]$
- $x[n] = e^{n^4}[\cos(\frac{\pi}{12}n)]u[n] - e^{n^4}[\cos(\frac{\pi}{12}n)]u[n-1]$

Problem 4. An LTI system is described by the difference equation

$$y[n] = \frac{9}{2}y[n-1] + \frac{5}{2}y[n-2] + x[n-1] + x[n]$$

(i). Find the system response $H(z) = Y(z)/X(z)$, and plot its pole-zero diagram.

(ii). Find the region of convergence of $H(z)$ when the system is

- anticausal, unstable.
- causal, unstable.
- causal, stable.

- noncausal, stable.

- Find the impulse response $h[n]$ of each of the above systems.

PROBLEM 5 (PROBLEM 5.36 IN OPPENHEIM, SCHAFER, BUCK). A causal linear time-invarient system has the system function

$$H(z) = \frac{(1 - 1.5z^{-1} - z^{-2})(1 + 0.9z^{-1})}{(1 - z^{-1})(1 + 0.7jz^{-1})(1 - 0.7jz^{-1})}$$

1. Write the difference equation that is satisfied by the input and the output of the system.

2. Plot the pole-zero diagram and indicate the region of convergence for the system function.

3. State whether the following are true or false about the system:

   - The system is stable.
   - The impulse response approaches a constant for large n.
   - The system has a stable and causal inverse.