ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 11 Introduction to Communication Systems
Homework 6 October 23, 2008

PROBLEM 1. Suppose we have a source S, which emits 5 symbols a, b, c, d, r.

1. Assume that we use the following code for encoding the source stream. What is the

Symbol	Code
a	01
b	10
c	11
d	000
r	001

encoder output for the input stream *a b r a c a d a b r a* ?

2. Assume that at the decoder you receive the bit stream {110001011011001}. What is the decoder output ?

3. Is it possible to have a uniquely decodable code for the present source S with length of the codewords restricted to be less than or equal to 2 ?

PROBLEM 2. 1. Consider a code for a source having 5 symbols, with lengths of codewords given by $l(s_1) = 3, l(s_2) = 3, l(s_3) = 3, l(s_4) = 2, l(s_5) = 2$. The code is shown in the table below. Is the Kraft's inequality satisfied ?

Symbol	Code
s_1	010
s_2	011
s_3	000
s_4	10
s_5	11

2. Is the code uniquely decodable ? If yes, explain.

PROBLEM 3. Construct the Huffman code for the following source S.

Symbol	Probability
s_1	0.20
s_2	0.15
s_3	0.25
s_4	0.25
s_5	0.15

PROBLEM 4. Suppose we have a source S with m symbols given by $\{1, 2, 3, \ldots, m\}$. Assume that the probability of symbol i is p_i.

1. Is it possible to construct a prefix free code with lengths of the codewords given by $l_i = \lceil \log_2(\frac{1}{p_i}) \rceil$?

2. If the answer to the above question is yes, then can you upper bound the average length of this code in terms of the entropy $H(S)$ of the source ?
   Hint: Use $x \leq \lceil x \rceil \leq x + 1$ for $x \geq 0$.