ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 6 Signal Processing for Communications
Homework 5. March 23, 2009

PROBLEM 1 (PROBLEM 5.1 IN THE BOOK). Consider the transformation $\mathcal{H}\{x[n]\} = nx[n]$. Does $\mathcal{H}$ define an LTI system?

PROBLEM 2 (PROBLEM 5.2 IN THE BOOK). Consider a discrete-time system $\mathcal{H}\{\cdot\}$. When the input is the signal $x[n] = \cos((2\pi/5)n)$, the output is $\mathcal{H}\{x[n]\} = \sin((\pi/2)n)$. Can the system be linear and time-invariant? Explain.

PROBLEM 3 (PROBLEM 5.3 IN THE BOOK). Consider the finite-support signal $h[n]$ defined as

$$h[n] = \begin{cases} 1 & \text{for } |n| \leq M \\ 0 & \text{otherwise} \end{cases}$$

1. Compute the signal $x[n] = h[n] * h[n]$ for $M = 2$ and sketch the result.
2. Compute the DTFT of $x[n]$, $X(e^{j\omega})$, and sketch its value in the interval $[0, \pi]$.
3. Give a qualitative description of how $|X(e^{j\omega})|$ changes as M grows.
4. Compute the signal $y[n] = x[n] * h[n]$ for $M = 2$ and sketch the result. For a general M, is the behavior of the sequence $y[n]$? (E.g. is it linear in n? Is it quadratic?)
5. Compute $Y(e^{j\omega})$ and sketch its value.

PROBLEM 4. Let $x[n]$ be a discrete-time sequence defined as

$$x[n] = \begin{cases} M - n & 0 \leq n \leq M \\ M + n & -M \leq n \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

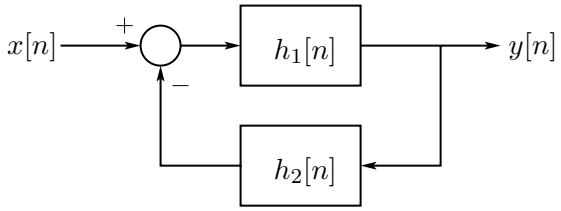
for some odd integer M.

1. Show that $x[n]$ can be expressed as the convolution of two discrete-time sequences $x_1[n]$ and $x_2[n]$.
2. Using the previous results, compute the DTFT of $x[n]$.

PROBLEM 5. Consider the feedback connection of two linear, time-invariant, discrete-time sysystems as shown in the figure. Given that

$$h_1[n] = \delta[n]$$
$$h_2[n] = (-2)^n u[n-1],$$

determine the impulse response $h[n]$ of the interconnected system. Are the systems represented by $h_1[n]$ and $h_2[n]$ stable? Is the overall system stable? Is it causal? Same question with $h_2[n] = (-2)^{n-1} u[n-1]$.



PROBLEM 6. Consider the cascade connection of two LTI systems. The system is characterized as follows:

$$H_1(e^{j\omega}) = \begin{cases} 1, & |w| < 0.5\pi \\ 0 & 0.5\pi \leq |\omega| < \pi. \end{cases}$$
$$y[n] = w[n] - w[n-1]$$

Determine the output $y[n]$ when $x[n] = \cos(0.6\pi n) + 3\delta[n-5] + 2$. (Hint: Plotting the spectrums of the signals and the impulse responses might help).