ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

Handout 5 Signal Processing for Communications

Homework 4. March 16, 2009

PROBLEM 1.

1. Write a Matlab function that takes four arguments:
   - frequency f,
   - amplitude a,
   - initial phase ϕ,
   - length N,

   and generates and plots $a\sin(2\pi f n + \phi), n = 0, \ldots, N-1$.

2. Plot the following sequences using your function for $n = 0, \ldots, 300$:
   - $f[n] = 2\sin(2\pi\frac{n}{15})$,
   - $g[n] = \sin(2\pi\frac{n}{25} + \frac{\pi}{3})$,
   - $f[n] + g[n]$,
   - $f[n]g[n]$.

   What are the periods of the above sequences? Use the Matlab function `subplot` to display all four sequences above in one window. Visually verify the periods of all four.

PROBLEM 2.

1. Write a Matlab function that takes as input a sequence $x[n]$ of length N, and an integer $L \geq N$ and does the following:
   - pad $x[n]$ with $L-N$ zeros, i.e., compute $y = (x[0], \ldots, x[N-1], 0, \ldots, 0)$,
   - compute the DFT of $y[n]$,
   - return both $y[n]$ and its DFT.

2. Using your DFT function, compute and plot (both the phases and the magnitudes of) the DFTs of the following sequences:
   - $x[n] = 1$, $n = 0, \ldots, N-1 = 9$. $L = 10, 100, 1000$.
   - $x[n] = \sin(\frac{2\pi}{8}n)$, $n = 0, \ldots, N-1 = 15$. $L = 16, 32, 64$.
   - $x[n] = \sin(\frac{2\pi}{8}n)$, $n = 0, \ldots, N-1 = 9$. $L = 10, 20, 30$.
   - $x[n] = \sin(\frac{2\pi}{5}n)$, $n = 0, \ldots, N-1 = 15$. $L = 16, 32, 64$.

   What do you observe?

3. Write a Matlab function that takes as input a sequence $X[k]$ of length N and an integer $L \leq N$ and computes

$$\hat{x}[n] = \sum_{k=0}^{L} X[k] e^{j\frac{2\pi}{N}kn}. \tag{1}$$

Suppose that $x \overset{\text{DFT}}{\longleftrightarrow} X$. Then, the sequence $\hat{x}[n]$ is an approximation of $x[n]$ obtained by taking the first L frequencies contained in x and ignoring the rest. In particular, $\hat{x} = x$ if $L = N$.

4. Using function you wrote in part 3, compute and plot (the magnitudes of) $\hat{x}[n]$ (given by (1)) for

   - $X[k] = 1, k = 0, \ldots, 7,\ L = 0, \ldots, 7.$
   - $X[k] = e^{-\frac{(k-128)^2}{100}}, k = 0, \ldots, 255.\ L = 0, \ldots, 255.$

   What happens as L increases?