ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
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Handout 7 Introduction to Communication Systems
Graded Homework October 9, 2008

PROBLEM 1. Suppose you are recording a song in a concert. Let $s(t)$ represent the song. Unfortunately there is a wild crowd at the concert. This adds noise to your recording. Let $n(t)$ represent the noise. Thus the total signal is given by $x(t) = s(t) + n(t)$. You store this song by sampling the total signal at $f_s = 8000$Hz. Assume that the noise signal $n(t)$ is a sinusoid of frequency 1000Hz, i.e., $n(t) = \sin(2000\pi t)$.

1. We have learnt in class that we can remove some noise by picking a moving average filter of some length, call it L_0 (at the cost of smoothing out the desired signal). It is natural to ask why not try to apply this procedure several times. In particular, assume that we take the signal $x(t)$ and send it through a moving average filter of length L_0 and then take the output and send it through a moving average filter of length L_0 again.
   (a) Is the overall system linear ?
   (b) Is the overall system time-invariant ?
   (c) Is the overall system causal?
   (d) Is the overall system stable ?
   (a) If you think that the overall system is linear and time-invariant, what is its impulse response? If you think it is either not linear or not time-invariant, think again.
2. Now suppose that instead of applying twice a moving average filter of length L_0 you apply one moving average filter of length $2L_0$. Is the output the same as the overall output in the previous question? In either case explain your answer briefly.
3. For the specific case given, is it possible to eliminate the noise by a moving average filter of length L by choosing a proper L? Explain.

PROBLEM 2. Consider the signal

$$x(t) = \sin(10\pi t)\cos(30\pi t)\sin(50\pi t) - \sin(5\pi t)\cos(20\pi t).$$

Suppose we sample the signal at $f_s = 32$Hz. We use the ideal interpolator to recover the signal.

1. What is the reconstructed signal?
2. What is the smallest sampling frequency so that the reconstructed signal is equal to the orignal signal ?

PROBLEM 3. Consider a system with impulse response given by

$$h_1[n] = \delta[n] - \delta[n-1], \quad \text{for all } n.$$

1. What is the output of the system if the input signal is $x[n] = 1$, for all $n \in \mathcal{Z}$.

2. What is the output of the system if the input signal is $x[n] = n$, for all $n \in \mathcal{Z}$.

Consider another system with impulse response given by

$$h_2[n] = 1, \quad \text{for all } n \geq 0.$$

1. What is the output of the system if the input signal is $x[n] = 1$, for all $n \geq 1$ and zero elsewhere.

2. What is the output of the system if the input signal is $x[n] = n$, for all $n \geq 0$ and zero elsewhere.

What is the output when we pass the signal $x[n] = n$, for all $n \geq 0$ and zero elsewhere, first through the system with impulse response $h_1[n]$ and then through the system with impulse response $h_2[n]$? Any comments ?

PROBLEM 4. Consider the continuous time signal

$$x(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 1, & 1 < t \leq 2 \\ 3 - t & 2 < t \leq 3 \\ 0 & t \notin [0, 3] \end{cases}$$

Sketch the signals $x(t)$, $-x(t)$, $x(-t)$, $x(3t)$, $x((t-3)/3)$, $x^2(t)$, $x(t^2)$.