ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 3 Signal Processing for Communications
Homework 3. Due: March 16, 2009 March 9, 2009

PROBLEM 1. Let $x[n]$, $n = 0, \dots, N-1$ be a complex signal. Prove the following properties of its DFT:

1. If $x[n] \longleftrightarrow X[k]$, then $x^*[-n \mod N] \longleftrightarrow X^*[k]$.
2. If $x[n] \longleftrightarrow X[k]$, then $X[n] \longleftrightarrow Nx[-k \mod N]$.

Let $x[n]$ be a real sequence and let $x_e[n]$ and $x_o[n]$ denote its even and odd parts. That is,

$$x_e[n] = \frac{1}{2}(x[n] + x[-n \mod N]) \qquad x_o[n] = \frac{1}{2}(x[n] - x[-n \mod N]).$$

Prove the following:

3. $X[k] = X^*[-k \mod N]$. What does this imply on $\Re\{X[k]\}$ and $\Im\{X[k]\}$?
4. $x_e[n] \longleftrightarrow \Re\{X[k]\}$ and $x_o[n] \longleftrightarrow j\Im\{X[k]\}$

PROBLEM 2. Let $x_1[n]$ and $x_2[n]$ be length-N sequences. Also let

$$X_1[k] \circledast X_2[k] = \frac{1}{N}\sum_{l=1}^{N-1} X_1[l]X_2[k-l \mod N]$$

1. Show that $x_1[n]x_2[n] \longleftrightarrow X_1[k] \circledast X_2[k]$.
2. Show Parseval's relation: $\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N}\sum_{k=0}^{N-1} |X[k]|^2$.

PROBLEM 3. Let $\delta(t)$ denote the Dirac delta functional. Evaluate the following integrals:

1. $\int \delta(-t)f(t^2)dt$.
2. $\int \delta(2t-5)f(3t+2)dt$.
3. $\int \delta(-\frac{3t}{4}+1)f(-(\frac{t}{2})^2)dt$.
4. $\int \delta(\frac{2t}{3}+4)(f(t))^3dt$.
5. $\int \delta(-4t+5)(2f(2t)^2 + f(2t-1)+2)dt$.

PROBLEM 4 (EXERCISE 3.3 IN THE BOOK). Vector spaces and signals.

1. Show that the set of all ordered n-tuples $[a_1, a_2, \dots, a_n]$ with the natural definition for the sum:

$$[a_1, a_2, \dots, a_n] + [b_1, b_2, \dots, b_n] = [a_1+b_1, a_2+b_2, \dots, a_n+b_n]$$

and the multiplication by a scalar:

$$\alpha[a_1, a_2, \dots, a_n] = [\alpha a_1, \alpha a_2, \dots, \alpha a_n]$$

form a vector space. Give its dimension and find a basis.

2. Show that the set of signals of the form $y(x) = a\cos(x) + b\sin(x)$ (for arbitrary a, b), with the usual addition and multiplication by a scalar, form a vector space. Give its dimension and find a basis.

3. Are the four diagonals of a cube orthogonal?

4. Express the discrete-time impulse $\delta[n]$ in terms of the discrete-time unit step $u[n]$ and conversely.

5. Show that any function $f(t)$ can be written as the sum of an odd and an even function, i.e., $f(t) = f_o(t) + f_e(t)$ where $f_o(-t) = -f_o(t)$ and $f_e(-t) = f_e(t)$.

Problem 5. Compute the DTFT of the following signals. Plot the magnitude of each DTFT.

1. $x[n] = n(u[n+3] - u[n-3])$

2. $x[n] = n\frac{1}{3}^{|n|}u[-n-2]$

3. $x[n] = 2^n \sin(\frac{\pi}{4}n)u[-n]$

4. $x[n] = \sin(\frac{\pi}{2}n + \frac{\pi}{6}) + \cos(n)$

5. $x[n] = \cos(\frac{\pi}{4}n)\sin(\frac{2\pi}{5}n)$