

# ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

**Handout 2**  
Homework 2.

Signal Processing for Communications  
March 2, 2009

PROBLEM 1. Let  $v[n]$ ,  $n = 0, \dots, 2N - 1$  be a real sequence of length- $2N$  with  $2N$ -point DFT  $V[k]$ . Let  $g_1[n]$  and  $g_2[n]$  be two length- $N$  sequences consisting of the even and the odd indices of  $v[n]$ . That is,

$$g_1[n] = v[2n], \quad g_2[n] = v[2n + 1] \quad 0 \leq n \leq N - 1$$

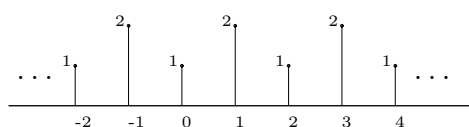
Let  $G_1[k]$  and  $G_2[k]$  denote the  $N$ -point DFT of  $g_1$  and  $g_2$  respectively. Define a new complex sequence

$$x[n] = g_1[n] + jg_2[n].$$

1. Find  $X^*[-k \bmod N]$ , in terms of  $G_1[k]$  and  $G_2[k]$ .
2. Express  $G_1[k]$  and  $G_2[k]$ , in terms of  $X[k]$ .
3. Express  $V[k]$  in terms of  $G_1[k]$  and  $G_2[k]$ .

PROBLEM 2 (PROBLEM 8.2 IN OPPENHEIM, SCHAFER, BUCK). Suppose  $\tilde{x}[n]$  is a periodic sequence with period  $N$ . Then  $\tilde{x}[n]$  is also periodic with period  $3N$ . Let  $\tilde{X}[k]$  denote the DFS coefficients of  $\tilde{x}[n]$  considered as a periodic sequence with period  $N$ , and let  $\tilde{X}_3[n]$  denote the DFS coefficients of  $\tilde{x}[n]$  considered as a periodic sequence with period  $3N$ .

1. Express  $\tilde{X}_3[k]$  in terms of  $\tilde{X}[k]$ .
2. By explicitly calculating  $\tilde{X}[k]$  and  $\tilde{X}_3[k]$ , verify your result in Part 1 when  $\tilde{x}[n]$  is as given in the figure below.



PROBLEM 3 (PROBLEM 8.5 IN OPPENHEIM, SCHAFER, BUCK). Compute the DFS coefficients of the periodic sequences below.

1.  $\tilde{x}_1[n] = e^{-3(n \bmod 15)}$ .
2.  $\tilde{x}_2[n] = \begin{cases} 1, & \text{if } n \text{ even} \\ -1, & \text{if } n \text{ odd} \end{cases}$
3.  $\tilde{x}_3[n] = \begin{cases} \tilde{x}_2[\frac{n}{2}], & \text{if } n \text{ even} \\ \tilde{x}_2[\frac{n-1}{2}], & \text{if } n \text{ odd} \end{cases}$

PROBLEM 4 (PROBLEM 8.47 IN OPPENHEIM, SCHAFER, BUCK). Let  $x[n]$ ,  $n = 0, \dots, 7$  be a length-8 sequence and let  $X[k]$ ,  $k = 0, \dots, 7$  be its DFT.

1. Evaluate

$$\left( \frac{1}{8} \sum_{k=0}^7 X[k] e^{j\left(\frac{2\pi}{8}\right)kn} \right) \Big|_{n=9}$$

in terms of  $x[n]$ .

2. Let  $w[n]$   $n = 0, \dots, 3$  be a length-4 sequence, and let  $W[k]$  be its length-4 DFT. If  $W[k] = X[k] + X[k + 4]$ ,  $k = 0, \dots, 3$ , express  $w[n]$  in terms of  $x[n]$ .
3. Let  $y[n]$   $n = 0, \dots, 7$  be a length-8 sequence, and let  $Y[k]$  be its DFT. If

$$Y[k] = \begin{cases} 2X[k] & \text{for } k = 0, 2, 4, 6, \\ 0 & \text{for } k = 1, 3, 5, 7, \end{cases}$$

express  $y[n]$  in terms of  $x[n]$ . (*Hint:* Start by expressing  $Y[k]$  in terms of  $X[k]$ .)

PROBLEM 5 (EXERCISE 4.9 IN THE BOOK). Consider the following infinite non-periodic discrete time signal:

$$x[n] = \begin{cases} 0, & \text{if } n < 0 \\ 1, & \text{if } 0 \leq n \leq a \\ 0, & \text{if } n > a \end{cases}$$

1. Compute its DTFT  $X(e^{j\omega})$ .

We want to visualize the magnitude of  $X(e^{j\omega})$  using a numerical package (for instance, Matlab). Most numeric packages cannot handle continuous sequences such as  $X(e^{j\omega})$ ; therefore we need to consider only a finite number of points for the spectrum.

2. Plot 10,000 points of one period of  $|X(e^{j\omega})|$  (from 0 to  $2\omega$ ) for  $a = 20$ .

The DTFT is mostly a theoretical analysis tool, and in many cases, we will compute the DFT. Moreover, for obvious reasons, numeric computation programs as, Matlab, only compute the DFT. Recall that in Matlab we use the Fast Fourier Transform (FFT), an efficient algorithm to compute the DFT.

3. Generate a finite sequence  $x_1[n]$  of length  $N = 30$  such that  $x_1[n] = x[n]$  for  $n = 1, \dots, N$ . Compute its DFT and plot its magnitude. Compare it with the plot obtained in (2).
4. Repeat now for different values of  $N = 50, 100, 1000$ . What can you conclude?