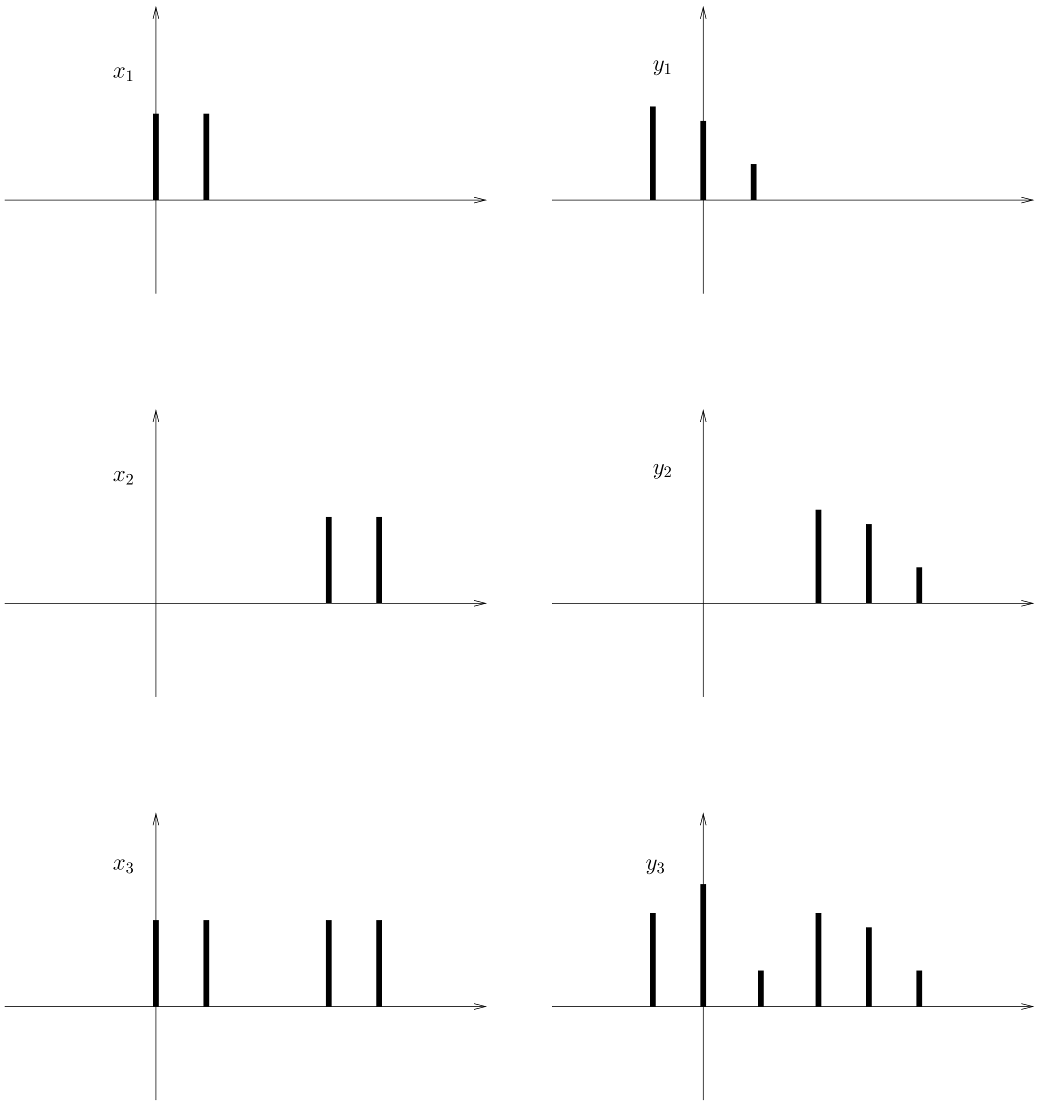
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

Handout 2 Introduction to Communication Systems
Homework 2 September 25, 2008

PROBLEM 1. Consider a system S and imagine that you use three test signals $x_1(n), x_2(n)$ and $x_3(n)$ to study its behaviour. Let $y_1(n), y_2(n)$ and $y_3(n)$ be the corresponding outputs.



On the basis of the correspondence between the input and the output signals, tick the appropriate answer to the following questions.

(a) Is the system S causal ? (Yes, No, Cannot Say)

(b) Is the system S linear ? (Yes, No, Cannot Say)

(c) Is the system S time-invariant ? (Yes, No, Cannot Say)

PROBLEM 2. Suppose

$$h[n] = \begin{cases} \frac{1}{n}, & n \geq 1 \\ 0, & \text{else} \end{cases}$$

Is a system with impulse response given by $h[n]$ stable ? What can you say about the stability of the system if the impulse response is $h[n] = \frac{1}{n^2}$ for $n \geq 1$ and is zero elsewhere? In both cases, if you think that the system is not stable, then find a bounded input signal so that the output signal is unbounded.

PROBLEM 3. Consider the signal

$$x(n) = \begin{cases} |\frac{n}{2} + \frac{1}{n}| & \text{if } -2 \leq n < 0, \text{and } 0 < n \leq 3 \\ 0 & \text{otherwise} \end{cases}.$$

Sketch the following signals:

(a) $x(n)$

(b) $x(n+3)$

(c) $x(2-n)\delta(n-1)$

(d) $x(n-1) \star \delta(3-n)$

PROBLEM 4.

(a) Consider a filter H with impulse response

$$h(n) = \begin{cases} |\frac{1}{n}| & \text{if } -2 \leq n < 0 \text{ and } 0 < n \leq 2 \\ 0 & \text{otherwise} \end{cases}.$$

Sketch the output of H when the input signal is given by

$$x(n) = \begin{cases} 3 - \frac{1}{n} & 0 < n < 3 \\ 0 & \text{otherwise} \end{cases}.$$

(b) Consider the system H obtained by cascading two filters H_1 and H_2 of impulse response $h_1(n)$and $h_2(n)$ respectively. What is the impulse response of H? Is H also a filter? What happens if we swap H_1 and H_2?

PROBLEM 5. Suppose we have a system S with impulse response

$$h[n] = \begin{cases} \alpha^n, & n \geq 0 \\ 0, & \text{else} \end{cases}$$

where $\alpha < 1$.

1. Suppose we give to the system an input signal $x[n] = \sin(\omega n)$. We want to compute the output $y[n]$ of the system S. We will proceed as follows. It is convenient to work in the domain of complex numbers. Recall that $a = x + jy$ is a complex number with $x, y \in \mathbb{R}$ and $j^2 = -1$. We denote by $\text{Im}(a)$ the imaginary part of the complex number a. So here $\text{Im}(a) = y$. Also $|a| = \sqrt{x^2 + y^2}$ denotes the magnitude of a. Recall also that $e^{j\omega} = \cos\omega + j\sin\omega$.

(a) How can you represent the signal $x[n]$ as a complex signal ?

(b) Prove that $\text{Im}(\cdot)$ is a linear function.

(c) Compute the output of the system S if the input signal is $x[n] = e^{j\omega n}$. To evaluate the output you will find the following identity to be useful:

$$1 + a + a^2 + a^3 + \cdots = \frac{1}{1-a} \qquad \text{if } |a| < 1$$

Note that the summation above is over infinite terms.

(d) Using the answers to the above questions can you compute the output signal $y[n]$?

2. What can you say about the amplitude of the output signal $y[n]$ for $\omega = 0$?

3. What can you say about the amplitude of the output signal $y[n]$ for $\omega = \pi$?

4. What happens to the amplitude in both cases above as $\alpha \to 1$ from below ?