ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 1 Signal Processing for Communications
Homework 1 February 23, 2008

PROBLEM 1. Decide whether the following signals are periodic, and if so, find the period.

1. $x[n] = e^{j\frac{n}{\pi}}$
2. $x[n] = 2 + \sin(4\pi n) + 2\cos(3\pi n)$
3. $x[n] = 2\sin(5\pi n) + 3\sin(\sqrt{5}\pi n)$
4. $x[n] = 7\cos(2\pi n/7)\sin(2\pi n/5)$

PROBLEM 2. Consider a system with impulse response $h[n]$. Recall that fed with the input $x[n]$, the output $y[n]$ of such a system is given by

$$y[n] = x[n] * h[n].$$

(i). Decide whether a system with the following impulse responses is stable and/or causal.

1. $h[n] = -e^{|2n|}$
2. $h[n] = e^{2n}u[-n+1]$
3. $h[n] = (-1)^n u[3n]$
4. $h[n] = \frac{1}{3^n}u[n] + 4^n u[-n-2]$
5. $h[n] = \frac{1}{(n-1)^2}u[n-1]$

(ii). For the stable systems in part (i), compute $y[n]$ with the input $x[n] = u[n-2] - u[n-4]$.

(iii). Let $x[n] = u[n] - 2u[n-2] + u[n-4]$. Let $w[n]$ be the extension of the sequence $x[n]$ of period 4. That is,

$$w[n] = \sum_{k=-\infty}^{\infty} x[n-4k].$$

For the stable systems in part (i), compute $y[n]$ with the input $w[n]$.

PROBLEM 3 (EXERCISE 2.1 FROM THE BOOK).

(i). Let $s[n] = \frac{1}{2^n} + j\frac{1}{3^n}$. Compute $\sum_{n=1}^{\infty} s[n]$.

(ii). Same question with $s[n] = \left(\frac{j}{3}\right)^n$.

(iii). Characterize the set of complex numbers satisfying $z^* = z^{-1}$.

(iv). Find three complex numbers $\{z_0, z_1, z_2\}$ which satisfy $z_i^3 = 1, i = 1, 2, 3$.

(v). What is the following infinite product $\prod_{n=1}^{\infty} e^{j\pi/2^n}$?

PROBLEM 4 (EXERCISE 3.2 FROM THE BOOK). Let $\{\mathbf{x}^{(k)}\}_{k=0,\ldots,N-1}$ be a basis for a subspace S. Prove that any vector $\mathbf{z} \in S$ is *uniquely* represented in this basis. (*Hint: prove by contradiction.*)

PROBLEM 5 (EXERCISE 4.1 FROM THE BOOK). Derive the formula for the DFT of the length-N signal $x[n] = \cos((2\pi/N)Ln + \phi)$.

PROBLEM 6 (EXERCISE 4.2 FROM THE BOOK). Compute the DFT of the length-4 signal $x[n] = \{a, b, c, d\}$. For which values of $\{a, b, c, d\}$ is the DFT real?