ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 18 Signal Processing for Communications
Homework 11. Due: May 18, 2009 May 11, 2009

PROBLEM 1. Let $x(t)$ $t \in \mathbb{R}$ be a complex signal with Fourier transform $X(j\Omega)$. Prove the following:

1. $x(t-\tau) \overset{FT}{\longleftrightarrow} e^{-j\tau\Omega}X(j\Omega)$.
2. $X(jt) \overset{FT}{\longleftrightarrow} 2\pi x(-\Omega)$. *Hint*: $e^{jt\Omega_0} \overset{FT}{\longleftrightarrow} 2\pi\delta(\Omega - \Omega_0)$.
3. $x(at) \overset{FT}{\longleftrightarrow} \frac{1}{a}X(j\frac{\Omega}{a})$.

PROBLEM 2. Let $x_1(t)$ and $x_2(t)$, $t \in \mathbb{R}$ be complex signals, and let "$*$" denote the convolution operator. Show that

1. Show that
$$(x_1 * x_2)(t) \overset{FT}{\longleftrightarrow} X_1(j\Omega)X_2(j\Omega).$$
2. Show Parseval's relation:
$$\int_{-\infty}^{\infty} |x_1(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X_1(j\Omega)|^2 d\Omega.$$

PROBLEM 3. Consider Figure 9.13 on p. 260 in the book (otherwise, see the first figure in Problem 5). The input and the output to the system is given by

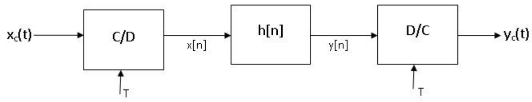
$$Y_c(j\Omega) = H(e^{j\Omega T_s})X_c(j\Omega),$$

where X_c and Y_c are the FTs of the input $x_c(t)$ and output $y_c(t)$ respectively. We want the system to output the derivative of its input, i.e., $y_c(t) = x_c'(t)$.

1. Show that $x'(t) \overset{FT}{\longleftrightarrow} j\Omega X(j\Omega)$. Check this for $x(t) = \cos(\Omega_0 t)$.
2. Following Figure 9.13, we assume that $y[n] = x'[n]$. Show that
$$h[n] = \begin{cases} 0 & \text{for } n = 0 \\ \frac{(-1)^n}{nT_s} & \text{otherwise} \end{cases}.$$

PROBLEM 4. Problem 9.4 in the book.

PROBLEM 5. Consider the following system:



Suppose that the frequency response of the input is given in Figure 1. Determine the largest possible value of T and find and plot $H(e^{j\omega})$, the DTFT of $h[n]$ for which

$$Y_c(j\Omega) = \begin{cases} |\Omega| X_c(j\Omega) & 1000\pi < |\Omega| < 2000\pi \\ 0 & \text{otherwise} \end{cases}.$$

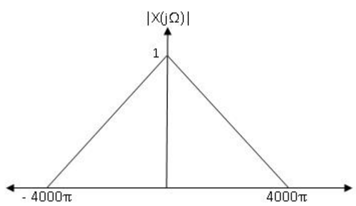


Figure 1: