ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE

School of Computer and Communication Sciences

Handout 1 Modern Coding Theory

Please choose at least 3 out of the following 7 problems. April 15, 2008

PROBLEM 1. Consider $A(G, w)$, the number of codewords of weight w which are contained in the graph G. Consider an (l, r)-regular ensemble of LDPC codes of length n.

- Compute the expected value of $A(G, w)$ for $w = 1, 2, 3$ and n tending to infinity as a function of l and r.
- Comment on your results. Are there some special values of l or r that give you qualitatively different results than other values?
- BONUS: Consider the case $l = 2$. Is it true that $A(G, w = 2)$ converges in distribution to a Poisson?

PROBLEM 2. Problem 3.5

PROBLEM 3. Problem 3.6

PROBLEM 4. Consider the ensemble with degree distribution pair $(\lambda(x) = \frac{1}{2}x + \frac{1}{2}x^4, \rho(x) = x^5)$ and transmission over the BEC. What is its rate, what is its threshold under BP, an what is its threshold under MAP decoding according to the Maxwell construction?

PROBLEM 5. In class we only considered the BP algorithm for general BMS channels. But quantized decoders are important for practical applications. The simplest quantized decoder is the Gallager algorithm A, which you can find described in Section 4.5.1. Implement the density evolution recursion for the $(4, 5)$-regular ensemble and the Gallager algorithm A and transmission over the BSC. What is the threshold?

PROBLEM 6. Compute the threshold of the $(4, 5)$-regular ensemble and the BSC channel under the BP decoder.

HINT: It is quite time-consuming to implement density evolution. But there is an easy shortcut. The easiest way to determine the threshold is by means of a "population dynamics" approach. This works as follows. Start with a population of lets say m independent samples from the initial density (this is the initial density of log-likelihood ratios for a BSC with parameter ϵ). Now create from this a population of m samples after the first half iteration in the following way. To get one sample, pick $r - 1$ of the original samples independently and uniformly at random and combine these samples according to the BP message-passing equations. Repeat this procedure m times. Next do the equivalent for the next half iteration, but now using the message-passing rules at the variable nodes and also including samples that correspond to the received values. This simulates one iteration of density evolution. In C or Mathematica or MatLab only a few lines of code should be necessary.

PROBLEM 7. Compute the threshold of the $(4, 5)$-regular ensemble and the BSC cannel under the BP decoder by using the "extemes of information combining" ideas. More precisely, to get a lower bound on the threshold assume at check nodes that all incoming densities are BEC densities and at variable nodes assume that incoming messages are from the BSC family. Make the opposite assumptions in order to derive an upper bound on the threshold. If you have done the previous problem you should be able to confirm that the real threshold lies between these two bounds.