PROBLEM 1. Implement the density evolution equations for the $(4, 5)$-regular ensemble and the BSC channel under Gallager A decoder. What is the threshold for this ensemble? What is the rate? How does the threshold compare to the channel parameter that we should be able to tolerate according to Shannon?

PROBLEM 2. Compute the thresholds of the $(2, 3)$-regular and the $(4, 5)$-regular ensemble and the BSC channel under the BP decoder. [We will see next time how to the determine the threshold of the first case analytically.]

HINT: It is quite time-consuming to implement density evolution. But there is an easy shortcut. The easiest way to determine the threshold is by means of a "population dynamics" approach. This works as follows. Start with a population of lets say m independent samples from the initial density (this is the initial density of log-likelihood ratios for a BSC with parameter ϵ). Now create from this a population of m samples after the first half iteration in the following way. To get one sample, pick $r - 1$ of the original samples independently and uniformly at random and combine these samples according to the BP message-passing equations. Repeat this procedure m times. Next do the equivalent for the next half iteration, but now using the message-passing rules at the variable nodes and also including samples that correspond to the received values. This simulates one iteration of density evolution. In C or Mathematica or MatLab only a few lines of code should be necessary.