ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 1 Modern Coding Theory
Homework 2 February 27, 2008

PROBLEM 1. Problem 1.19 from MCT.

PROBLEM 2. In class we have seen how we can factorize the bit-MAP decoding problem for a binary linear block code. If the corresponding factor graph is a tree then we can apply the general message passing decoder to accomplish bit-MAP decoding. In this problem we will see an alternative way of accomplishing bit-MAP decoding by means of message passing. It is based on the trellis representation of a linear block code which you have seen in Problem 1.

Assume you are given a binary linear block code C of length n and dimension k defined by its parity check matrix H. Let X be chosen uniformly at random from C and let Y be the result of transmitting X over the binary symmetric channel with parameter ϵ. We are interested in bit-MAP decoding, i.e., we are interested in

$$\hat{x}_i(Y) = \text{argmax}_{x_i \in \{0,1\}} p(X_i \mid Y).$$

Associate to every codeword x a *state* vector, call it σ. Recall that x has length n, $x = (x_1, \dots, x_n)$. We have $\sigma = (\sigma_0, \dots, \sigma_n)$, where $\sigma_i \in F_2^{n-k}$. We have $\sigma_0 = (0, \dots, 0)$ and for $i \in [n]$,

$$\sigma_i = \sigma_{i-1} + x_i h_i,$$

where h_i is the i-th column of H. This state is exactly the sequence of vertices which x goes through in the trellis representation of Problem 1.

(i) Derive a factorization of $\hat{x}_i(Y)$ by writing $p(X, Y)$ as $\sum_{\Sigma} p(X, Y, \Sigma)$.

(ii) Draw the corresponding factor graph which associates one variable node to each variable and one function node to each factor. Is this factor graph a tree?

(iii) Consider the Hamming code. Assume that the received word is $(1, 0, 0, 1, 1, 0, 1)$. Apply the standard message-passing algorithm (sum product) to this decoding problem.

1. (iv) Is this algorithm optimal? What is the complexity?

(v) Can you think of an alternative way of implementing the algorithm directly on the trellis derived in Problem 1?

Note: The algorithm which you derived is called the BCJR algorithm. If instead of using the sum-product algorithm you employ the min-sum algorithm you get the Viterbi algorithm, which optimizes the word error probability instead of the bit error probability. This exercise shows that in this way *any* binary linear block code can be decoded optimaly by a message-passing decoder. So what is the catch?