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MIDTERM EXAM

Closed book, one page of notes allowed  
Time limit: 110 minutes

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Print Name

Problem	Score
#1	/ 25
#2	/ 25
#3	/ 25
#4	/ 25
Total	/ 100

**Problem 1** (*Warmup Problem: Background Material*)

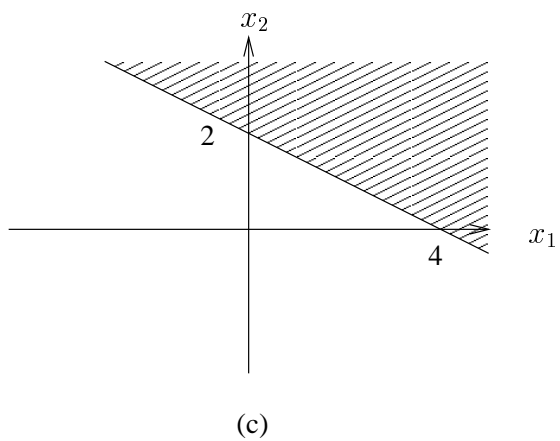
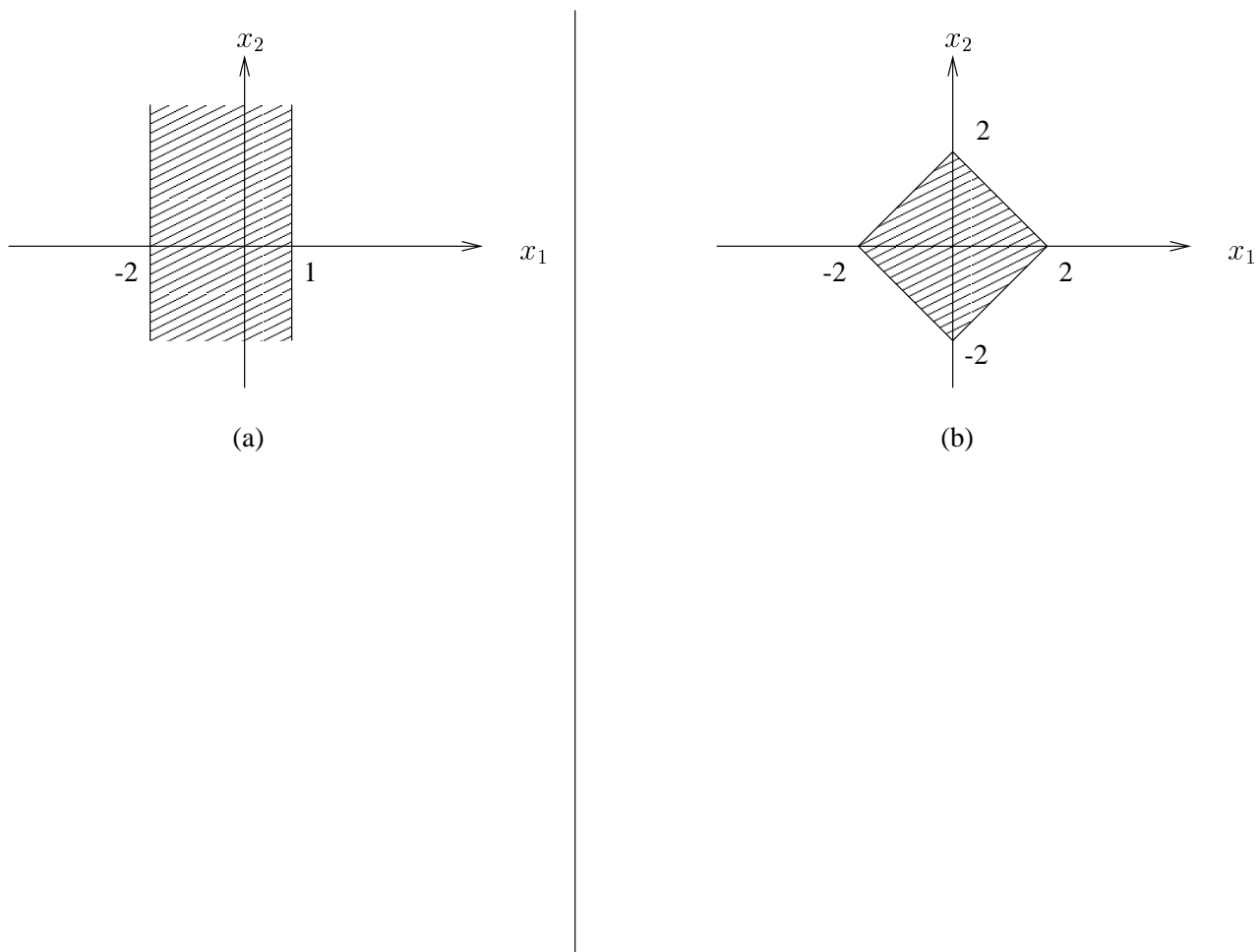
- (a) Assume that  $X_1$  and  $X_2$  are independent random variables and that they are uniformly distributed in the interval  $[0, 1]$ . What is the probability that  $X_1 + X_2 < 1$  and  $X_2 \geq \frac{1}{2}$ ?

- (b) Let  $\phi(t) = A \frac{\sin \frac{\pi}{T} t}{\frac{\pi}{T} t}$ ,  $t \in \mathbb{R}$ . Sketch  $\phi(t)$  and its Fourier transform  $\phi_{\mathcal{F}}(f)$ . Label your figures appropriately.

- (c) Determine  $A$  so that  $\phi(t)$  has unit energy.

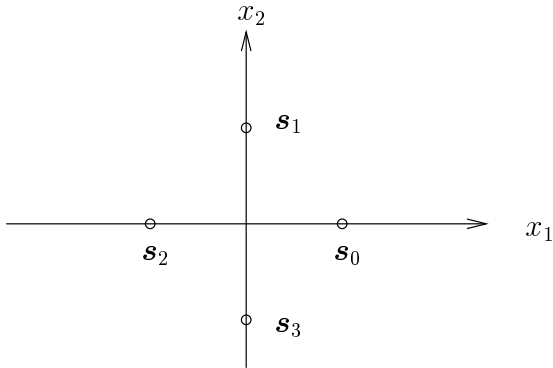
### Problem 2

Let  $\mathbf{X} \sim \mathcal{N}(0, \sigma^2 I_2)$ . For each of the three figures below, express the probability that  $\mathbf{X}$  lies in the shaded region. You may use the  $Q$ -function when appropriate.

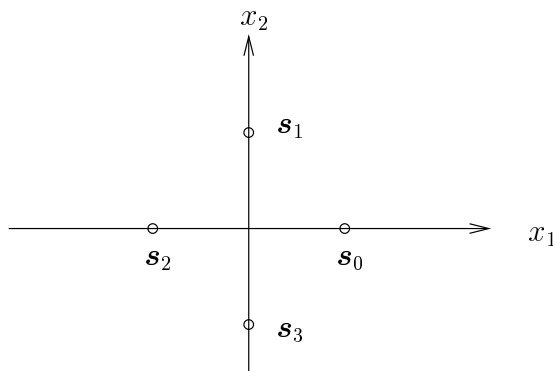


**Problem 3**

Let  $H \in \{0, 1, 2, 3\}$  and assume that when  $H = i$  you transmit the signal  $\mathbf{s}_i$  shown in the figure. Under  $H = i$ , the receiver observes  $\mathbf{Y} = \mathbf{s}_i + \mathbf{Z}$ .



- (a) In the above figure, draw the decoding regions assuming that  $\mathbf{Z} \sim \mathcal{N}(0, \sigma^2 I_2)$  and that  $P_H(i) = 1/4 \forall i$ .
- (b) In the figure below, draw the decoding regions (qualitatively) assuming  $\mathbf{Z} \sim \mathcal{N}(0, \sigma^2 I)$  and  $P_H(0) = P_H(2) > P_H(1) = P_H(3)$ . Justify your answer.



- (c) Assume again that  $P_H(i) = 1/4 \forall i$  and that  $\mathbf{Z} \sim \mathcal{N}(0, K)$ , where  $K = \begin{pmatrix} \sigma^2 & 0 \\ 0 & 4\sigma^2 \end{pmatrix}$ . How do you decode now? Justify your answer.

#### Problem 4

You are taking a multiple choice exam. Question number 5 allows for two possible answers. According to your first impression, answer 1 is correct with probability  $1/4$  and answer 2 is correct with probability  $3/4$ .

You would like to maximize your chance of giving the correct answer and you decide to have a look at what your left and right neighbors have to say.

The left neighbor has answered  $\hat{H}_L = 1$ . He is an excellent student who has a record of being correct 90% of the time.

The right neighbor has answered  $\hat{H}_R = 2$ . He is a weaker student who is correct 70% of the time.

- (a) You decide to use your first impression as a prior and to consider  $\hat{H}_L$  and  $\hat{H}_R$  as observations. Describe the corresponding hypothesis testing problem. **Note:** Be concise and clear. The point here is to show that you know what constitutes relevant information.

- (b) What is your answer  $\hat{H}$ ? Justify it.