ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

Principles of Digital Communications: Summer Semester 2007

Assignment date: April 5, 2007
Due date: April 19, 2007

Homework 4

Problem 1. Lecture notes Problem 12 *(Q-PSK decision regions)* of Section 2.9

Problem 2. Lecture notes Problem 14 *(Multiple choice exam)* of Section 2.9

Problem 3. Lecture notes Problem 15 *(QAM with erasure)* of Section 2.9

Problem 4. *(Dice Tossing)*

You have two dices, one fair and one loaded (*truqué*). A friend told you that the loaded dice produces a 6 with probability $\frac{1}{4}$, and the other values with uniform probabilities. You do not know a priori which one is fair or which one is loaded. You pick with uniform probabilities one of the two dices, and perform N consecutive tosses (*lancés*) with the dice you have chosen. Let

$$Y = (Y_1, \cdots, Y_N)$$

be the sequence of numbers observed.

(a) Based on the sequence of observations Y, find the decision rule to determine whether the dice you have chosen is loaded. Your decision rule should maximize the probability of correct decision.

(b) Identify a compact sufficient statistic for this hypothesis testing problem, call it S. Justify your answer. [Hint: $S \in \mathbb{N}$.]

(c) Find the Bhattacharyya bound on the probability of error. You can either work with the observation $(Y_1, \ldots, Y_N)$ or with $(Z_1, \ldots, Z_N)$, where Z_i indicates whether the ith observation is a six or not, or you can work with S. In some cases you may find it useful to know that $\sum_{i=0}^{N} \binom{N}{i} x^i = (1+x)^N$ for $N \in \mathbb{N}$. In other cases the following may be useful: $\sum_{Y_1, Y_2, \ldots, Y_N} \prod_{i=1}^{N} f(Y_i) = \left(\sum_{Y_1} f(Y_1) \right)^N$.