ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Principles of Digital Communications: Summer Semester 2007

Assignment date: March 29, 2007
Due date: April 5, 2007

Homework 3

Problem 1. *(Rayleigh distribution)*

Let X and Y be two independent, zero-mean, unit-variance, Gaussian random variables: $(X, Y) = \mathcal{N}(0, \boldsymbol{I}_2)$. Let R and Θ be the corresponding polar coordinates, i.e., $X = R\cos\Theta$ and $Y = R\sin\Theta$. Find the probability density functions $f_{R,\Theta}$, f_R, and f_Θ.

Hint: Try do solve this problem without looking up formulas. First write down the expression of $f_{R,\Theta}$ as a function of $f_{X,Y}$ assuming that you have a linear transformation of the kind $(X, Y)^T = A(R, \Theta)^T$ for some 2×2 invertible matrix A. See the Appendix of the lecture notes, Chapter 2, if you don't know how to do this from memory using the hints given in class. Recall that $\det A$ equals $\frac{1}{\det A^{-1}}$, which means that you may work with the determinant of A or with that of A^{-1}, whichever is more convenient. Next, instead of A use the Jacobian J of the transformation that maps R, Θ into X, Y. The Jacobian is the matrix that maps $(dR, d\Theta)^T$ into $(dX, dY)^T$.

Problem 2. Lecture notes Problem 7 *(Transformation of random vectors)* of Section 2.9

Problem 3. Lecture notes Problem 8 *(Theorem of irrelevance and Sufficient statistics)* of Section 2.9

Problem 4. Lecture notes Problem 9 of Section 2.9