ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Principles of Digital Communications: Assignment date: June 14, 2007
Summer Semester 2007 Due date: June 21, 2007

# Homework 11

**Problem 1.** Lecture notes Problem 5 *(Intersymbol Interference)* of Section 6.6
**Typo:** The first equation should read $Y_i = S_i + Z_i$.

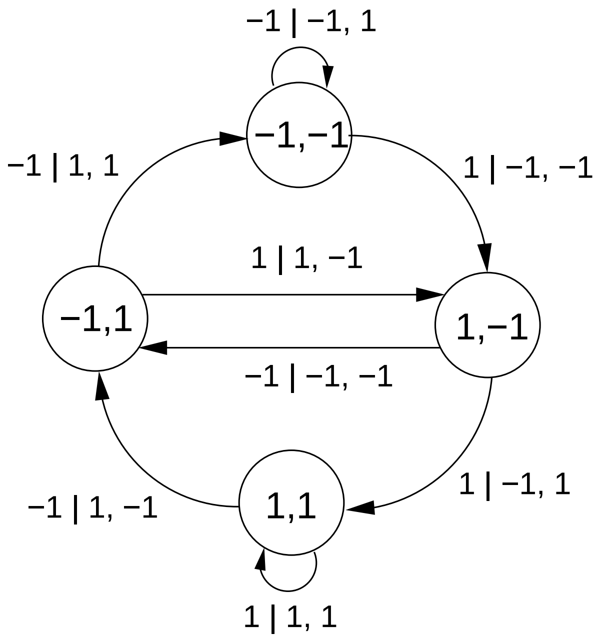
**Problem 2.** Lecture notes Problem 6 *(Linear Transformations)* of Section 6.6

**Problem 3.** *(Convolutional Encoder, Decoder and Error Probability Analysis)*

Consider a channel, where a transmitter wants to send a sequence $\{D_j\}$ taking values in $\{-1, +1\}$, for $j = 0, 1, 2, \cdots, k-1$. This sequence is encoded using a convolutional encoder. The channel adds white Gaussian noise to the transmitted signal. If we let $X_j$ denote the transmitted value, then, the received value is: $Y_j = X_j + Z_j$, where $\{Z_j\}$ is a sequence of i.i.d. zero-mean Gaussian random variables with variance $\frac{N_0}{2}$. The receiver has to decide which sequence was transmitted using the optimal decoding rule.

a. Convolutional Encoder

Consider the convolutional encoder corresponding to the finite state machine drawn below. The transitions are labeled by $D_j | X_{2j}, X_{2j+1}$, and the states by $D_{j-1}, D_{j-2}$. We assume that the initial content of the memory is $(1, 1)$.




1. What is the rate of this encoder?
2. Sketch the filter (composed of shift registers and multipliers) corresponding to this finite state machine. How many shift registers do you need?
3. Draw a section of the trellis representing this encoder.

b. Viterbi Decoder

Let $X_j^i$ denote the output of the convolutional encoder at time $j$ when we transmit hypothesis $i$, $i = 0, \cdots, m-1$, where $m$ is the number of different hypotheses.

Assume that the received vector is $\bar{Y} = (Y_1, Y_2, Y_3, Y_4, Y_5, Y_6) = (-1, -3, -2, 0, 2, 3)$. It is the task of the receiver to decide which hypothesis $i$ was chosen or, equivalently, which vector $\bar{X}^i = (X_1^i, X_2^i, X_3^i, X_4^i, X_5^i, X_6^i)$ was transmitted.

1. Use the Viterbi algorithm to find the most probable transmitted vector $\bar{X}^i$.

c. Performance Analysis

1. Suppose that this code is decoded using the Viterbi algorithm. Draw the detour flow graph, and label the edges by the input weight using the symbol $I$, and the output weight using the symbol $D$.
2. Considering the following generating function

$$T(I, D) = \frac{ID^4}{1 - 3ID},$$

What is the value of

$$\sum_{i,d} i a(i,d) e^{-\frac{d}{2N_0}},$$

where $a(i,d)$ is the number of detours with $i$ bit errors and $d$ channel errors? First compute this expression, then give an interpretation in terms of probability of error of this quantity.

*Hints: Recall that the generating function is defined as* $T(I,D) = \sum_{i,d} a(i,d) D^d I^i$. *You may also use the formula* $\sum_{k=1}^{\infty} k q^{k-1} = \frac{1}{(1-q)^2}$ *if* $|q| < 1$.