Final Exam

Last Name	First Name

Problem	Points	out of
Problem 1		21
Problem 2		28
Problem 3		28
Problem 4		21
Bonus		2
Total		100

Remarks.

- This is a three-hour exam.
- This is a closed-book exam. No supporting material is allowed.
- The problems are not in order of difficulty.
- A correct answer does not guarantee full credit and a wrong answer does not guarantee loss of credit. You should concisely indicate your reasoning and show all relevant work. The grade on each problem is based on our judgment of your level of understanding as reflected by what you have written.
- Try to do side calculations on a separate sheet and to report on this exam sheet well-organized solutions. If we can't read it, we can't grade it.
- There are 14 sub-questions. Each question is worth 7 points. You get an extra 2 points bonus for well-organized solutions.
- Ask if you think that a question is not clear.

Problem 1 (21 Points)

The process of storing and retrieving binary data on a thin-film disk may be modeled as transmitting binary symbols across an additive white Gaussian noise channel where the noise Z has a variance that depends on the transmitted (stored) binary symbol S. The noise has the following input-dependent density:

$$f_Z(z) = \begin{cases} \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{z^2}{2\sigma_1^2}} & \text{if } S = 1 \\ \frac{1}{\sqrt{2\pi\sigma_0^2}} e^{-\frac{z^2}{2\sigma_0^2}} & \text{if } S = 0, \end{cases}$$

where $\sigma_1 > \sigma_0$. The channel inputs are equally likely.

(a) On the same graph, plot the two possible output probability density functions. Indicate, qualitatively, the decision regions.

(b) Determine the optimal receiver in terms of σ_1 and σ_0. (Use the back of the previous page for details.)

(c) Write an expression for the error probability P_e as a function of σ_0 and σ_1.

Problem 2 (Cioffi) (28 Points)

The signal set

$$s_0(t) = \text{sinc}^2(t)$$
$$s_1(t) = \sqrt{2}\text{sinc}^2(t)\cos(4\pi t)$$

is used to communicate across an AWGN channel of power spectral density $\frac{N_0}{2}$.

(a) Find the Fourier transforms of the above signals and plot them.

(b) Sketch a block diagram of a ML receiver for the above signal set.

(c) Determine its error probability of your receiver assuming that $s_0(t)$ and $s_1(t)$ are equally likely.

(d) If you keep the same receiver, but use $s_0(t)$ with probability $\frac{1}{3}$ and $s_1(t)$ with probability $\frac{2}{3}$, does the error probability increase, decrease, or remain the same? Justify your answer.

Problem 3 (28 Points)

Consider using the signal set

$$s_i(t) = s_i\phi(t), \quad i = 0, 1, \ldots, m-1,$$

where $\phi(t)$ is a unit-energy waveform, $s_i \in \{\pm\frac{d}{2}, \pm\frac{3}{2}d, \ldots, \pm\frac{m-1}{2}d\}$, and $m \geq 2$ is an even integer.

(a) Assuming that all signals are equally likely, determine the average energy $\mathcal{E}_s$ as a function of m. Hint: $\sum_{i=0}^{n} i^2 = \frac{n}{6} + \frac{n^2}{2} + \frac{n^3}{3}$. Note: If you prefer you may determine an approximation of the average energy by assuming that $S(t) = S\phi(t)$ and S is a continuous random variable which is uniformly distributed in the interval $[-\frac{m}{2}d, \frac{m}{2}d]$.

(b) Draw a block diagram for the ML receiver, assuming that the channel is AWGN with power spectral density $\frac{N_0}{2}$.

(c) Give an expression for the error probability.

(d) For large values of m, the probability of error is essentially independent of m but the energy is not. Let k be the number of bits you send every time you transmit $s_i(t)$ for some i, and rewrite $\mathcal{E}_s$ as a function of k. For large values of k, how does the energy behaves when k increases by 1?

Problem 4 (21 Points)

A bandpass signal $x(t)$ may be written as $x(t) = \sqrt{2}\Re\{x_E(t)e^{j2\pi f_0 t}\}$, where $x_E(t)$ is the baseband equivalent of $x(t)$.

(a) Show that a signal $x(t)$ can also be written as $x(t) = a(t)\cos[2\pi f_0 t + \theta(t)]$ and describe $a(t)$ and $\theta(t)$ in terms of $x_E(t)$. Interpret this result.

(b) Show that the signal $x(t)$ can also be written as $x(t) = x_{EI}(t)\cos 2\pi f_0 t - x_{EQ}(t)\ \sin(2\pi f_0 t)$, and describe $x_{EI}(t)$ and $x_{EQ}(t)$ in terms of $x_E(t)$. (This shows how you can obtain $x(t)$ without doing complex-valued operations.)

(c) Find the baseband equivalent of the signal $x(t) = A(t)\cos(2\pi f_0 t + \varphi)$, where $A(t)$ is a real-valued lowpass signal. Hint: You may find it easier to *guess* an answer and verify that it is correct.