Final Exam

Last Name	First Name

Problem	Points	out of
Problem 1		35
Problem 2		25
Problem 3		30
Problem 4		10
Total		100

Remarks.

- You have 180 minutes to complete the exam.
- This is a closed-book exam.
- There are 4 problems on the exam.
- The problems are not necessarily in order of difficulty.
- A correct answer does not guarantee full credit and a wrong answer does not guarantee loss of credit. You should concisely indicate your reasoning and show all relevant work. The grade on each problem is based on our judgment of your level of understanding as reflected by what you have written.
- Try to do side calculations on a separate sheet and to report on this exam sheet well-organized solutions. If we can't read it, we can't grade it.
- If you don't understand a problem, please ask.

Problem 1 (35 Points)

One of the two signals $s_0 = -1, s_1 = 1$ is transmitted over the channel shown on the left of Figure 1.

The two noise random variables Z_1 and Z_2 are statistically independent of the transmitted signal and of each other.

Their density functions are

$$f_{Z_1}(\alpha) = f_{Z_2}(\alpha) = \frac{1}{2}\, e^{-|\alpha|}.$$

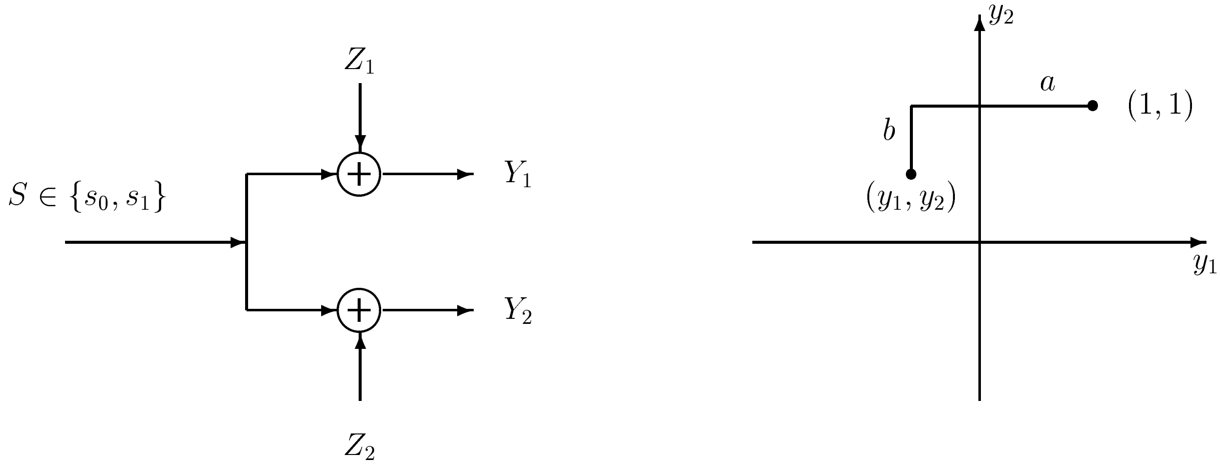

Figure 1:

(a) Derive a maximum likelihood decision rule.

(b) Describe the maximum likelihood decision regions in the (y_1, y_2) plane. You get extra points if you describe the "Either Choice" regions, i.e., the regions in which it does not matter if you decide for s_0 or for s_1. Hint: Use geometric reasoning and the fact that for a point (y_1, y_2) as shown on the right of Figure 1, $|\, y_1 - 1 \,| + |\, y_2 - 1 \,| = a + b$.

(c) A receiver decides that s_1 was transmitted if and only if $(y_1 + y_2) > 0$. Does this receiver minimize the error probability for equally likely messages?

(d) What is the error probability for the receiver in (c)? Hint: if $W = Z_1, + Z_2$ then $f_W(\omega) = \frac{e^{-\omega}}{4}(1 + \omega)$ for $w > 0$.

(e) Could you have derived f_W as in (d) ? If yes, say how but omit detailed calculations.

Problem 2 (25 Points)

One of two signals shown in Figure 2 is transmitted over the additive white Gaussian noise channel. There is no bandwidth constraint and either signal is selected with probability 1/2.

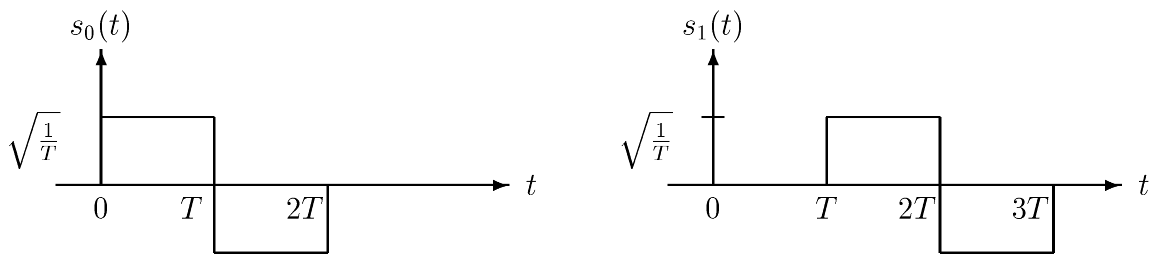

Figure 2:

(a) Draw a block diagram of a maximum likelihood receiver. Be as specific as you can. You get extra points for using the smallest possible number of filters and/or correlators.

(b) Determine the error probability in terms of the Q-function, assuming that the power spectral density of the noise is $\frac{N_0}{2} = 5 \ [\frac{\text{W}}{Hz}]$.

Problem 3 (30 Points)

Consider the transmitter shown in Figure 3, when $\ldots D_{-i}, D_i, D_{i+1}, \ldots$ is a sequence of independent and uniformly distributed random variables taking value in $\{\pm 1\}$.

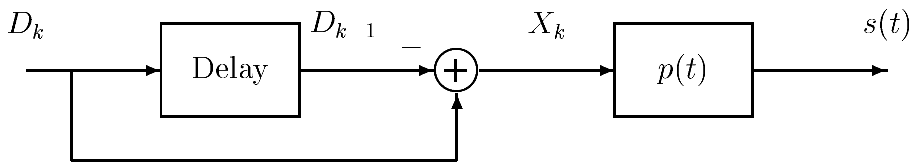

Figure 3:

The transmitted signal is

$$s(t) = \sum_{i=-\infty}^{\infty} X_i p(t - iT)$$

$$X_i = D_i - D_{i-1}$$

$$p(t) = 1_{\left[-\frac{T}{2}, \frac{T}{2}\right]}(t).$$

(a) Determine $R_X[k] = E[X_{i+k}X_i]$.

(b) Determine $R_p(\tau) = \frac{1}{T}\int_{-\infty}^{\infty} p(t+\tau)p(t)dt$.

(c) Determine and plot the Fourier transform of $p(t)$.

Problem 4 (10 Points)

In this problem we neglect noise and consider the situation in which we transmit a signal $X(t)$ and receive

$$R(t) = \sum_i \alpha_i X(t - \tau_i).$$

Show that the baseband equivalent relationship is

$$R_E(t) = \sum_i \beta_i \, X_E(t - \tau_i).$$

Express β_i explicitly.