Principles of Digital Communications
Prof. Bixio Rimoldi

EPFL Summer Semester 2001
July 13, 2001

Final Exam

Last Name	First Name

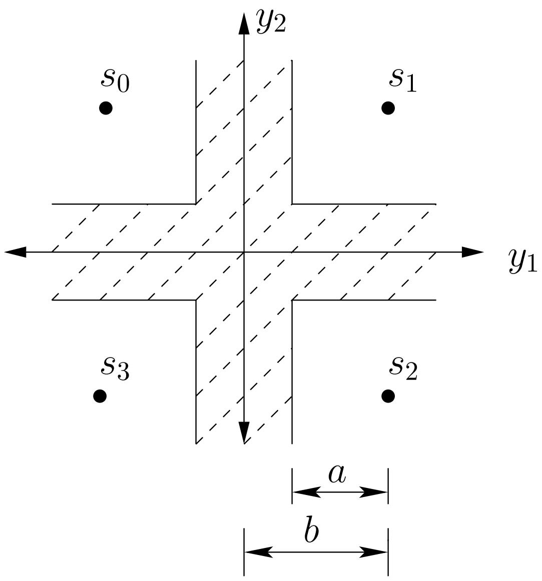
Problem	Points	out of
Problem 1		15
Problem 2		25
Problem 3		15
Problem 4		20
Problem 5		20
Total		95

Remarks.

- You have 150 minutes to complete the exam.
- This is a closed-book exam. You may use one double-sided sheet of notes.
- There are 5 problems on the exam.
- The problems are not necessarily in order of difficulty.
- A correct answer does not guarantee full credit and a wrong answer does not guarantee loss of credit. You should concisely indicate your reasoning and show all relevant work. The grade on each problem is based on our judgment of your level of understanding as reflected by what you have written.
- Try to do side calculations on a separate sheet and to report on this exam sheet well-organized solutions. If we can't read it, we can't grade it.
- If you don't understand a problem, please ask.

Problem 1 (15 Points) *(Modified QPSK demodulator)*

Consider a QAM receiver designed to output a special symbol (called "erasure" and denoted by δ) whenever the observation falls in the shaded area shown in the figure below.



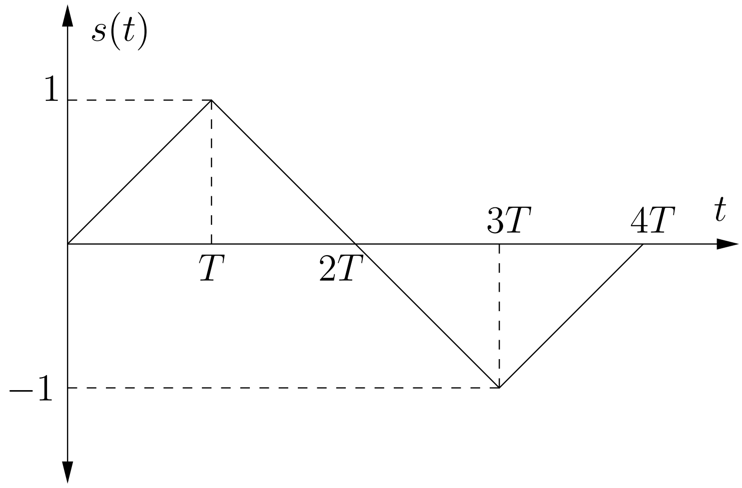
Assume that $\mathbf{s}_0$ is transmitted and that $\mathbf{Y} = \mathbf{s}_0 + \mathbf{N}$ is received where $\mathbf{N} \sim \mathcal{N}(0, \sigma^2 I_2)$. Let P_{0i}, $i = 0, 1, 2, 3$ be the probability that the receiver outputs $\hat{H} = i$ and let $P_{0\delta}$ be the probability that it outputs δ. Determine P_{00}, P_{01}, P_{02}, P_{03} and $P_{0\delta}$. (*Write on the back of this page if you need more space*).

Problem 2 (25 Points)

Consider the following equiprobable binary hypothesis testing problem specified by:

$$\begin{aligned} H = 0 &: \; Y(t) = s(t) + N(t) \\ H = 1 &: \; Y(t) = N(t) \end{aligned}$$

where $N(t)$ is AWGN (Additive White Gaussian Noise) of power spectral density $N_0/2$ and $s(t)$ is the signal shown below.



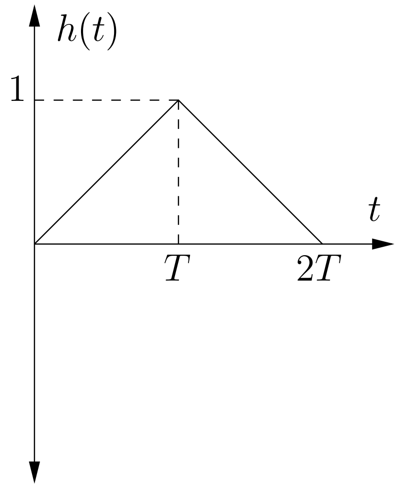
(a) First consider a receiver that only observes $Y(t_0)$ for some fixed t_0. Choose the best possible value of t_0 and describe the receiver that minimizes the error probablity based on the observation $Y(t_0)$.

(b) Determine the error probability for the receiver you described in *(a)*.

(c) Describe the maximum-likelihood receiver for the observable $Y(t)\,,\; t \in \mathcal{R}\,.$

(d) Determine the error probability for the receiver you described in *(c)*.

(e) Can you realize your receiver in *(c)* using a filter with impulse response $h(t)$ as shown below?



Problem 3 (15 Points) *(Antenna Array)*

Consider an L-element antenna array as shown in the figure below.

Let $u(t)\beta_i$ be the (complex-valued baseband equivalent) signal transmitted at antenna element i, $i = 1, 2, \ldots, L$ (according to some indexing which is irrelevant here) and let

$$v(t) = \sum_{i=1}^{L} u(t - \tau_D)\beta_i \alpha_i$$

(plus noise) be the sum-signal at the receiver antenna, where α_i is the path strength for the signal transmitted at antenna element i and τ_D is the (common) path delay.

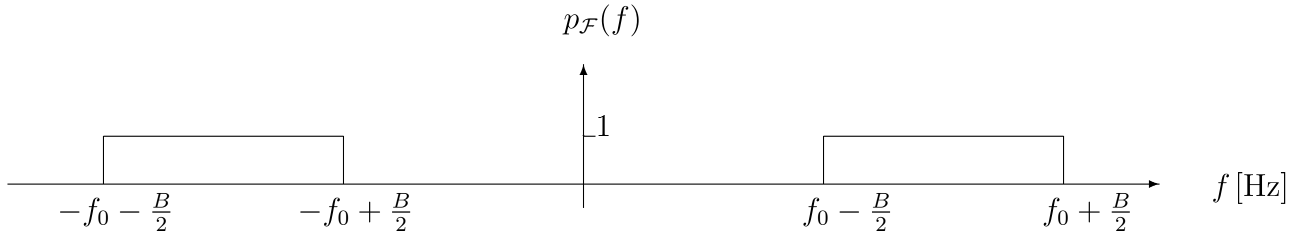
(a) Choose the vector $\beta = (\beta_1, \beta_2, \ldots, \beta_L)^T$ that maximizes the signal energy at the receiver, subject to the constraint $\| \beta \| = 1$. The signal energy is defined as $E_v = \int |v(t)|^2 dt$. **Hint** Use the Cauchy-Schwarz inequality: for any two vectors $\mathbf{a}$ and $\mathbf{b}$ in $\mathbb{C}^n$, $|\langle \mathbf{a}, \mathbf{b} \rangle|^2 \leq \| \mathbf{a} \|^2 \| \mathbf{b} \|^2$ with equality iff $\mathbf{a}$ and $\mathbf{b}$ are linearly dependent.

(b) Let $u(t) = \sqrt{E_u}\phi(t)$ where $\phi(t)$ has unit energy. Determine the received signal power as a function of L when β is selected as in (a) and $\alpha = (\alpha, \alpha, \ldots, \alpha)^T$ for some complex number α.

(c) (Optional) In the above problem the received energy grows monotonically with L while the transmit energy is constant. Does this violate energy conservation or some other fundamental low of physics? Hint: an antenna array is not an isotropic antenna (i.e. an antenna that sends the same energy in all directions).

Problem 4 (20 Points) *(Nyquist)*

Consider a pulse $p(t)$ defined via its Fourier transform $p_{\mathcal{F}}(t)$ as follows:



(a) What is the expression for $p(t)$? (If you can't determine a mathematical expression, you may draw $p(t)$ qualitatively).

(b) Determine the constant c so that $\psi(t) = cp(t)$ has unit energy.

(c) Assume that $f_0 - \frac{B}{2} = B$ and consider the infinite set of functions $\cdots,\ \psi(t+T)$, $\psi(t)$, $\psi(t-T)$, $\psi(t-2T)$, $\cdots$. Do they form an orthonormal set for $T = \frac{1}{2B}$? (Explain).

(d) (optional) Determine all possible values of $f_0 - \frac{B}{2}$ so that $\cdots,\ \psi(t+T),\ \psi(t)$, $\psi(t-T),\ \psi(t-2T),\ \cdots$ forms an orthonormal set.

Problem 5 (20 Points) *(Bluetooth Radio System)*

(i) In which band does *Bluetooth* operate?

(ii) On what multiple access scheme is *Bluetooth* system based?

(iii) What is the modulation scheme used by *Bluetooth*?

(iv) What is the difference between master and slave in a piconet?

(v) Does *Bluetooth* allow a slave to talk to another slave? (Explain).

(vi) How is it decided who is the master of a piconet?

(vii) What makes it possible to mitigate the interference? (e.g. interference created by co-located piconets).