Signal Processing for Communications
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Handout # xx, Monday, October 22, 2007

Homework Set # 4

Problem 1

Prove the following properties of the DTFT:

- $x[-n] \stackrel{\text{DTFT}}{\longleftrightarrow} X(e^{-j\omega})$
- If $x[n]$ is real, then $X(e^{j\omega}) = X^*(e^{-j\omega})$.
- If $x[n]$ is real and symmetric ($x[n] = x[-n]$), then $X(e^{j\omega})$ is real.
- If $x[n]$ is real and antisymmetric ($x[n] = -x[-n]$), then $X(e^{j\omega})$ is purely imaginary.

Problem 2

(a) Compute the DTFT of $x[n] = n2^{-n}u[n]$, using the method seen in Homework 1 for geometric series of the form $\sum_n nr^n$.

(b) Compute again the DTFT of $x[n]$, but now use the DTFT property

$$nx[n] \stackrel{\text{DTFT}}{\longleftrightarrow} j\frac{\mathrm{d}}{\mathrm{d}\omega}X(e^{j\omega})$$

seen in class. Verify that the two methods give the same result.

Problem 3 (DFT and DTFT)

Let us consider a sequence $\overline{x}[n]$ with DTFT $X(e^{j\omega})$. If the sequence has finite support of length N or less, it can be recovered from $X[k]$, the N-point DFT of the corresponding finite-length sequence $x[n]$. Hence the DTFT of $\overline{x}[n]$ is uniquely determined by the N-point DFT of $x[n]$. Show that:

$$X(e^{j\omega}) = \sum_{n=0}^{N-1} x[n]e^{-j\omega n} = \frac{1-e^{-j\omega N}}{N}\sum_{k=0}^{N-1}\frac{X[k]}{1-e^{j(2\pi k/N-\omega)}}$$

Problem 4 (Linearity and Time-Invariance)

(a) True or false: A BIBO stable, causal, linear system must be time-invariant. Give a proof if you believe it's true; give a counterexample if you believe it's false.

(b) True or false: If the (discrete-time) input to a linear, time-invariant system is periodic with period N, then its output must also be periodic with period N. Give a proof if you believe it's true; give a counterexample if you believe it's false.

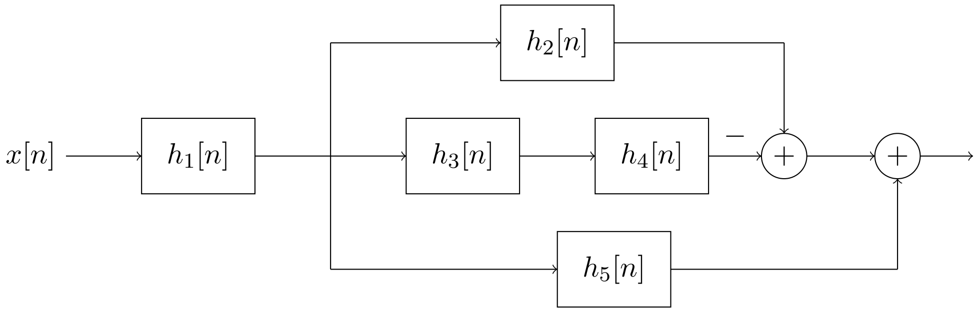

Figure 1: System for Problem 6

(c) Determine if the systems characterized by the following relations are (1) BIBO stable, (2) causal, (3) linear, and (4) time-invariant. Justify your answers with proofs or counter-examples. In all cases $x[n]$ is the input to the system and $y[n]$ is the output of the system.

1. $y[n] = g[n]x[n]$ with $g[n]$ given
2. $y[n] = \sum_{k=n_0}^{n} x[k]$, $n \geq n_0$
3. $y[n] = \sum_{k=n-n_0}^{n+n_0} x[k]$
4. $y[n] = ax[n] + b$

Problem 5

For an LTI system with impulse response $h[n]$, a signal $x[n]$ is called an *eigenfunction* of the system if $y[n] = (h * x)[n] = cx[n]$, where c is some constant.

Show that $x[n] = a^n$ is an eigenfunction of *any* linear time-invariant system by computing the convolution of $x[n]$ and the impulse response of the system, $h[n]$. Determine the corresponding eigenvalue c (which will be different for each system).

Problem 6

Consider the interconnection of systems shown on Figure 1.

(a) Express the overall impulse response h in terms if h_1, h_2, h_3, h_4, and h_5.

(b) Determine $h[n]$ when

$$\begin{aligned} h_1[n] &= \delta[n] - \frac{1}{2}\delta[n-1] \\ h_2[n] &= 2nu[n-1] \\ h_3[n] &= (n-2)u[n-1] \\ h_4[n] &= \delta[n-1] \\ h_5[n] &= nu[n]. \end{aligned}$$

(c) Determine the response of the system in the previous question when $x[n] = 2\delta[n] + \delta[n-2] - 3\delta[n-3]$.