Homework Set # 2

Problem 1

What is the period of the following sequence?

$$\tilde{x}[n] = 3 + \sin\left(\frac{2\pi}{5}n\right) + \cos\left(\frac{3\pi}{2}n\right)$$

Problem 2 (Discrete Fourier Transform)

Show how to compute the DFT of two even complex length-N sequences $x_1[n]$ and $x_2[n]$ performing only one length-N transformation. Follow the steps below:

(a) Build the auxiliary sequence $y(n) = W_N^n x_1[n] + x_2[n]$ ($W_N = e^{-j\frac{2\pi}{N}}$).

(b) Show that $Y[k] = X_1[k+1] + X_2[k]$.

(c) Using symmetry properties of the DFT, show that $Y[-k-1] = X_1[k] + X_2[k+1]$.

(d) Use the results of (b) and (c) to create a recursion to compute $X_1[k]$ and $X_2[k]$. Note that $X[0] = \sum_{n=0}^{N-1} x[n]$.

Problem 3

Compute the DFS coefficients of the periodic sequences below.

(a) $\tilde{x}[n] = e^{-2(n \mod 20)}$.

(b) $\tilde{x}[n] = \begin{cases} 1, & \text{for } n \text{ even} \\ -1, & \text{for } n \text{ odd} \end{cases}$

Problem 4 (MATLAB and FFT)

(a) Read the MATLAB help for the function `fft`.

(b) Compute (analytically) the DFT of the signal

$$x[n] = \sin\left(\frac{4\pi n}{16}\right), \quad n = 0, 1, \dots, N.$$

Now compute the DFT of this signal using `fft` and compare the results.

Problem 5

Let $\tilde{x}_1[n]$ be periodic with period $N = 50$, where one period is given by

$$\tilde{x}_1[n] = \begin{cases} ne^{-0.3n}, & 0 \le n \le 25 \\ 0, & 26 \le n \le 49 \end{cases}$$

and let $\tilde{x}_2[n]$ be periodic with period $N = 100$, where one period is given by

$$\tilde{x}_1[n] = \begin{cases} ne^{-0.3n}, & 0 \le n \le 25 \\ 0, & 26 \le n \le 99 \end{cases} .$$

These two periodic sequences differ in their periodicity but otherwise have equal nonzero samples.

(a) Find the DFS $\tilde{X}_1[k]$ of $\tilde{x}_1[n]$ (using the `fft` function) and plot samples (using the `stem` function) of its magnitude and angle versus k.

(b) Find the DFS $\tilde{X}_2[k]$ of $\tilde{x}_2[n]$ and plot samples of its magnitude and angle versus k.

(c) What is the difference between the above two DFS plots?

Consider now the periodic sequence $\tilde{x}_3[n]$ with period 100, obtained by concatenating two periods of $\tilde{x}_1[n]$. Clearly, $\tilde{x}_3[n]$ is different from $\tilde{x}_2[n]$, even though both of them are periodic with period 100.

(d) Find the DFS $\tilde{X}_3[k]$ of $\tilde{x}_3[n]$ and plot samples of its magnitude and angle versus k.

(e) What effect does the periodicity doubling have on the DFS?

Problem 6 (DFT AND DTFT)

Let the infinite sequence $x[n]$ be defined as

$$x[n] = \begin{cases} 7+n, & -6 \le n \le -1 \\ 6-n, & 0 \le n \le 5 \\ 0, & \text{otherwise,} \end{cases}$$

i.e., $x[n] = \{1, 2, \ldots, 6, 6, 5, \ldots, 1\}$ for $n = \{-6, -5, \ldots, 5, 5\}$.

(a) Write a MATLAB function `dtft` to compute the DTFT of an arbitrary sequence. The function should take as arguments a row vector x containing the sequence, a row vector with the set of frequencies ω on which the DTFT is to be evaluated, and a number indicating the index of the first sample of x.

For example, to compute the DTFT of the above sequence $x[n]$, you would write

```
>> X = dtft([1:6, 6:-1:1], linspace(0, 2*pi, 100), -6);
```

This would evaluate the DTFT of $x[n]$ on 100 equally spaced points between 0 and 2π.

(b) Use the function `dtft` to compute $X(e^{j\omega})$, the DTFT of $x[n]$, and plot (using the `plot` function) its magnitude and phase.

(c) Let $y[n]$ be the finite sequence of length 12 obtained by wrapping the "negative" parts of $x[n]$ to the positive axis, i.e., $x[n] = \{6, 5, \ldots, 1, 1, 2, \ldots, 6\}$, for $0 \leq n \leq 11$. Using the `fft` function, determine the DFT $Y[k]$ of $y[n]$. Plot (using the `stem` function) the magnitude and phase onto the magnitude and phase plots of (b), respectively (using the `hold` function), and verify that the DFT is indeed a sampled version of the DTFT.