Homework Set # 1

Mathematical Prerequisites

Problem 1 (Geometric Series)

(a) Consider the sequence $x[n] = a \cdot r^n$ for some real r. Let $S[n] = \sum_{k=0}^{n} x[k]$. You have seen in class that $S[n] = (1 - r^{n+1})/(1 - r)$. Find a closed-form expression for $\tilde{S}[n] = \sum_{k=0}^{n} akr^k$. (*Hint:* Write $\tilde{S}[n]$ as derivative of $S[n]$.)

(b) Give a closed-form expression for $\sum_{k=0}^{m} \exp(j2\pi k/n)$. What if $m = ln - 1$, $l \in \mathbb{N}$?

(c) Compute $\sum_{k=0}^{\infty} t[k]$, where $t[k] = 1/4^k + (1/3j)^k$.

(d) Compute $|\sum_{k=0}^{n-1} \exp(j\theta k)|$ and express the result without using exponentials.

Problem 2 (Complex Numbers)

(a) Express $|e^z|$ in terms of x and y, the real and imaginary parts of z.

(b) Find the real and imaginary parts of $\sin z$ and $\cos z$. Express y our answers in terms of regular and hyperbolic trigonometric functions.

(c) *Complex logarithm* Express $\log z$ in terms of the modulus and the argument of z. (*Hint:* Consider all solutions to the equation $e^{a+jb} = z$.)

Problem 3 (Linear Algebra)

(a) Compute the determinant of the following matrix.

$$A = \begin{bmatrix} 3 & 0 & -2 & 1 \\ 1 & 0 & 1 & 2 \\ 0 & 2 & 0 & 1 \\ -1 & -3 & 2 & 0 \end{bmatrix}.$$

(b) Show that the determinant of a matrix equals the product of the eigenvalues. (*Hint:* Imagine that the characteristic equation is factored into

$$\det(\mathbf{A} - \lambda\mathbf{I}) = (\lambda_1 - \lambda) \cdots (\lambda_n - \lambda) \qquad (1)$$

and make a clever choice of λ.)

(c) The *trace* of a matrix is defined as the sum of the diagonal elements. Show that the trace equals the sum of the eigenvalues, in two steps.

1. Find the coefficient of $(-\lambda)^{n-1}$ on the right-hand side of (1).

2. Look for all the terms in

$$\det(\mathbf{A} - \lambda\mathbf{I}) = \det \begin{bmatrix} a_{11} - \lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} - \lambda & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} - \lambda \end{bmatrix} \tag{2}$$

which involve $(-\lambda)^{n-1}$. Explain why they all come from the product down the main diagonal, and find the coefficient of $(-\lambda)^{n-1}$. Compare.

Introduction to MATLAB

Problem 4 (MATLAB)

The remaining exercises have to be done in MATLAB. The data/input files required to do the exercises can be found on the course website.

If you have not worked with MATLAB before, you can use the material in handout #4 to get a good introduction.

(a) Go through the following sections of the handout:

- 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 18 (planar plots only).

For this exercise you do not have to hand in anything :-).

Knowing the very basics, it is easy to learn about new functions in MATLAB. The excellent documentation can be accessed through the help menu, by executing the command `helpbrowser` or by using the `help` command for specific commands.

Problem 5 (Operators)

Suppose we have a matrix A given by

$$\begin{bmatrix} a + ib & c + id \\ e + if & g + ih \end{bmatrix}. \tag{3}$$

What will be the MATLAB output of the following commands: i) `A'` ii) `A.'`, iii) `fliplr(A)`, iv) `sum(A)`, v) `sum(A,2)`, vi) `A*A`, vii) `A.*A`. Hint: Have a look in the documentation for *arithmetic operations*.

Problem 6

Let $a(x) = \sum_{n=0}^{N-1} a_n x^n$ and $b(x) = \sum_{m=0}^{M-1} b_m x^m$ be two polynomials, and suppose they are represented in MATLAB as row vectors $\mathbf{a} = [a_0, \ldots, a_{N-1}]$ and $\mathbf{b} = [b_0, \ldots, b_{M-1}]$. Find an easy way to compute the coefficients of the polynomial $c(x) = a(x)b(x)$. (*Hint:* Write down the general formula for the coefficient c_n and conclude.)

Problem 7 (Sequences)

When plotting sequences in MATLAB, the basic `plot` command is not always the best thing to use. The problem is that by default it will create a continuous function based on the points you pass it. The way to solve this is using an extra argument to the plot function. To plot a sequence $a[n], n = 1, \ldots, N$, you can use for example

```
>> plot([1:N],a,'o');
```

A plotting command intended specifically for sequences is `stem`. Have a look in the MATLAB documentation for its use.

(a) Create a sequence $a[n] = \cos(2\pi n/15)$, $n = 1, \ldots, 45$. Give a stem-plot of the sequence.

(b) Construct a new sequence from $a[n]$ by selecting every fifth sample, provide a stem-plot.

(c) Complete the M-file `cshiftright.m`, such that it implements a function that takes a sequence $a[n]$ and a number N as arguments and that shifts $a[n]$ N samples to the right, circularly filling in the samples from the end of the sequence on the left. Provide the completed M-file.

(d) Circularly shift the sequence $a[n]$ by 8 samples to right. Provide a stem-plot.

Problem 8 (Audio)

MATLAB has builtin commands for handling audio. Take a look at the commands: `wavread` and `soundsc`.

(a) Load the audio file `handel.wav` using

```
>> [data,fs] = wavread('handel.wav');
```

To listen to audio it is important to provide the sampling frequency used to record it. In this case it is provided as `fs` by the `wavread` command.

(b) Provide a stem-plot of samples 1 till 100.

(c) Create a new sequence by reversing the order of the audio sequence. Use `soundsc` to listen to the reversed sequence. Provide a stem-plot of samples 1 till 100 of the reversed sequence.

The `soundsc` will 'play' any sequence. The sequence does not necessarily have to be recorded audio.

(d) Create three sinusoidal sequences $a[n]$, $c[n]$ and $e[n]$, using `sin(f n/2e3)`, with f equal to 440 Hz, 523.25 Hz and 659.26 Hz respectively.

(e) Listen to the sequences $a[n]$, $a[n] + e[n]$ and $a[n] + c[n] + e[n]$. Use `soundsc(x,2e3)`.

(f) Give a plot of the first 300 samples of $a[n] + c[n] + e[n]$.

(g) Listen to the reversed sequences.

Problem 9 (Images)

MATLAB has builtin commands for loading and displaying images. Look in the documentation for `imread` and `imagesc`.

(a) The file `lena.jpg` is a 256 by 256 pixel grayscale image. Load and display `lena.jpg`. Hint: to display a grayscale image use `colormap(gray(256))`.

(b) Extract the top-left to bottom-right diagonal of the image. Provide a stem-plot of the extracted sequence.

By displaying a data sequence as an image we give an interpretation to the sequence. It makes sense to interpret the data in `lena.jpg` as an image. We might, however, just as well interpret this data as audio.

(c) Use `soundsc` to 'listen' to `lena.jpg`. Hint: To have all entries from a matrix `A` in a vector you can use e.g. `reshape(A,1,prod(size(A)))`, or simply `A(1:end)`.

The above exercise demonstrates that not all interpretations of a signal make sense.

In a similar way we can interpret some audio data as an image.

(d) Load `handel.wav`. Reshape the first 4096 samples of the data vector into a square matrix, *row first*. Display the absolute value of the matrix as a grayscale image. Provide a printout.

Also here we see that we have to choose the right interpretation in order to get results that make sense.