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## Two Additional Problems on the System Function

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### Problem 1

Consider the system function  $H(z) = \frac{1+2z^{-1}}{(1+\frac{7}{2}z^{-1}-2z^{-2})(1-2z^{-1})}$ . Find the possible corresponding impulse responses  $h[n]$ .

**Solution:** The poles are  $p_1 = 2$ ,  $p_2 = -4$ ,  $p_3 = \frac{1}{2}$ . After the partial fraction expansion, the system function is

$$H(z) = \frac{\frac{8}{27}}{1+4z^{-1}} + \frac{-\frac{5}{27}}{1-\frac{1}{2}z^{-1}} + \frac{\frac{8}{9}}{1-2z^{-1}}.$$

There is a causal system with ROC  $|z| > 4$  and impulse response

$$h[n] = \frac{8}{27}(-4)^n u[n] + \frac{-5}{27}\left(\frac{1}{2}\right)^n u[n] + \frac{8}{9}2^n u[n].$$

There is an anticausal system with ROC  $|z| < \frac{1}{2}$  and impulse response

$$h[n] = \frac{-8}{27}(-4)^n u[-n-1] + \frac{5}{27}\left(\frac{1}{2}\right)^n u[-n-1] + \frac{-8}{9}2^n u[-n-1].$$

There is a two-sided system with ROC  $2 < |z| < 4$  and impulse response

$$h[n] = \frac{-8}{27}(-4)^n u[-n-1] + \frac{-5}{27}\left(\frac{1}{2}\right)^n u[n] + \frac{8}{9}2^n u[n].$$

There is a two-sided and BIBO-stable system with ROC  $\frac{1}{2} < |z| < 2$  and impulse response

$$h[n] = \frac{-8}{27}(-4)^n u[-n-1] + \frac{-5}{27}\left(\frac{1}{2}\right)^n u[n] + \frac{-8}{9}2^n u[-n-1].$$

### Problem 2

Consider the system function  $H(z) = \frac{1+z^{-1}}{1-2z^{-1}+2z^{-2}}$ . Find the impulse response of a causal system that corresponds to this system function.

**Solution:** The poles are  $p_1 = 1 + j$  and  $p_2 = 1 - j$ . After the partial fraction expansion, the system function is

$$H(z) = \frac{\frac{1}{2} - j}{1 - (1 + j)z^{-1}} + \frac{\frac{1}{2} + j}{1 - (1 - j)z^{-1}}.$$

The impulse response is

$$h[n] = \left(\frac{1}{2} - j\right)(1 + j)^n u[n] + \left(\frac{1}{2} + j\right)(1 - j)^n u[n].$$