Advanced Digital Communications
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Homework 4

Problem 1

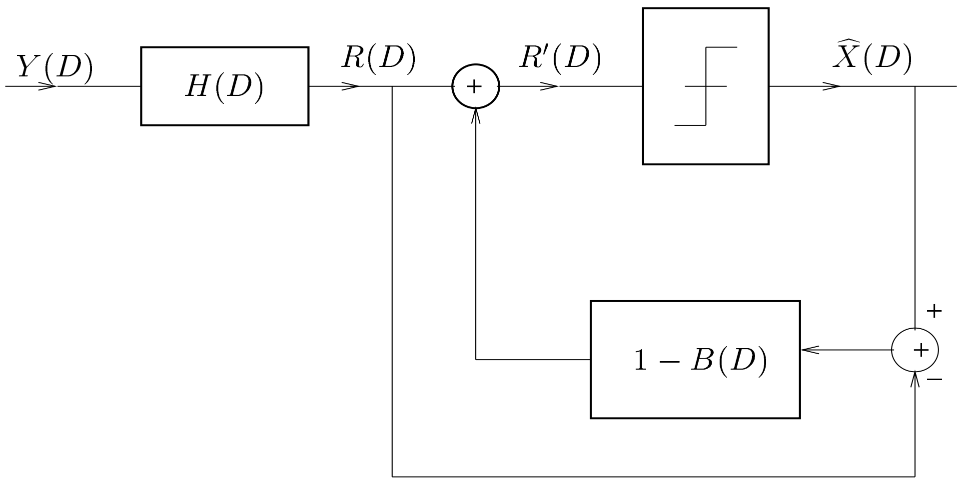

Figure 1: Block diagram for noise prediction DFE

Consider the discrete time model studied in class

$$Y(D) = ||p||Q(D)X(D) + Z(D),$$

where $S_x(D) = \mathcal{E}_x$, $S_z(D) = N_0 Q(D)$ with $Q(D) = Q^*(D^{-*})$. In class we derived the MMSE-DFE, but in this problem we consider a slightly different structure shown in Figure 1. As in class we consider perfect decision feedback, i.e., all past decisions are correct. Let

$$\begin{aligned} R(D) &= H(D)Y(D), \\ R'(D) &= R(D) + (1 - B(D))(X(D) - R(D)). \end{aligned}$$

We restrict $B(D)$ to be causal and monic, i.e.,

$$B(D) = 1 + \sum_{l=1}^{\infty} b_l D^l.$$

We choose $H(D)$ and $B(D)$ to minimize

$$\mathbb{E}\left(|x_k - r'_k|^2\right)$$

as we did in class.

(a) Find $H(D)$ in terms of $B(D)$ by using orthogonality principle.

(b) Set-up the prediction problem by proving that the error

$$E(D) = X(D) - R'(D) = B(D)X(D) - B(D)H(D)Y(D).$$

Use the solution of $H(D)$ in terms of $B(D)$ found in part (a) to show that

$$E(D) = B(D)U(D)$$

and find the expression for $U(D)$.

Show that

$$S_U(D) = \frac{N_0/||p||^2}{Q(D) + 1/SNR_{MFB}}.$$

Given this can you comment on the values of $H(D)$ and $B(D)$ with respect to the quantities derived in class. In particular, is the noise-prediction DFE the same as the MMSE-DFE derived in the class?

(c) If $B(D) = 1$, what does the structure in Figure 1 become?

Problem 2

Consider the channel

$$y(k) = ||p||x(k) * q(k) + z(k),$$

where $q(k) = \delta(k) + b\delta(k-1) + b\delta(k+1)$ and $z(k)$ is zero-mean Gaussian noise with power spectral density (PSD) $S_z(D) = N_0 Q(D)$. Assume that

$$b = \sqrt{\frac{N_0}{\mathcal{E}_x||p||^2}} = \frac{1}{2}.$$

Remember that by definition, $SNR_{MFB} = \frac{\mathcal{E}_x||p||^2}{N_0}$. In this problem, we consider a zero-forcing decision-feedback equalizer (**ZF-DFE**).

(a) Find the factorization

$$Q(D) = \nu_0 P_c(D) P_c^*(D^{-*}),$$

where $P_c(D)$ should be monic and causal.

(b) Find $B(D)$ and $W(D)$ for the ZF-DFE.