Homework 3, part2

Problem 3

Let $\{x_n\}$ is a sequence of wide sense stationary processes with autocorrelation

$$r_x(k) = \begin{cases} (\frac{2}{3})^{|k|} & \text{if } k \neq 0 \\ \frac{23}{28} & \text{if } k = 0 \end{cases}$$

(a) You have seen one step prediction in class where we use the realization of the sequence up to time k to estimate x_{k+1}. Let the sequence is given to you up to time k. Use $\{x_n\}_{n=-\infty}^{k}$ and find the MMSE linear estimator for x_{k+2} (**two step prediction**), i. e., $\hat{x}_{k+2} = \sum_{m=2}^{\infty} a_m x_{k+2-m}$.

(b) Use the results you have seen in class to make MMSE linear estimator for x_{k+1} in terms of $\{x_n\}_{n=-\infty}^{k}$, i. e., find the optimal $\{b_m\}$ in $\hat{x}_{k+1} = \sum_{m=1}^{\infty} b_m x_{k+1-m}$.

(c) Consider the sequence $\{y_n\}$ which is defined as $y_n = x_n$, for $n \leq k$, and $y_{k+1} = \hat{x}_{k+1}$, which you found in (b). Use the same argument as part (b) and find the MMSE linear estimator for y_{k+2}, i. e.,

$$\hat{y}_{k+2} = \sum_{m=1}^{\infty} c_m y_{k+2-m}.$$

(d) Using the result of (c), replace y_{k+1} with $\sum_{m=1}^{\infty} b_m x_{k+1-m}$, and find the coefficients of $\hat{y}_{k+2} = \sum_{m=2}^{\infty} d_m x_{k+2-m}$, in terms of $\{b_m\}$ and $\{c_m\}$. Compare $\{d_m\}$ to $\{a_m\}$ and comment.

Problem 4

Let $\{X_k\}$, $\{Y_k\}_{-\infty}^{\infty}$ be two correlated wide-sense stationary processes. Let $R_{XY}(D)$ denote the D transform of $\mathbb{E}[X_n Y_{n-k}]$ and $R_{YY}(D)$ denote the D transform of $\mathbb{E}[Y_n Y_{n-k}]$. Let $\{Z_{1k}\}, \{Z_{2k}\}$ be two independent sequences of i.i.d. Gaussian random variables, also independent from $\{X_k\}, \{Y_k\}$, with $Z_{1k} \sim \mathcal{N}(0, \sigma_1^2)$, $Z_{2k} \sim \mathcal{N}(0, \sigma_2^2)$. Let us define the processes $\{U_{1k}\}$ and $\{U_{2k}\}$ by

$$U_{1k} = Y_k + Z_{1k}$$
$$U_{2k} = Y_k + Z_{2k}.$$

Find the $W_{opt}(D)$ for estimating X_k from $\{U_{1k}, U_{2k}\}_{-\infty}^{\infty}$ when

(i) $\sigma_1 = 0$, $\sigma_2 = 2$,

(ii) $\sigma_1 = 1$, $\sigma_2 = 2$.

(iii) Explain the two results.