Homework 1 - part 2

Problem 5
If $\mathbf{Z} = (Z_1, Z_2)$ is a Gaussian random vector, then by definition, Z_1 and Z_2 are Gaussian random variables. However, the contrary is not true in general. Construct a simple example showing that Z_1 and Z_2 being Gaussian random variables does not imply that Z_1 and Z_2 are jointly Gaussian.

Problem 6
Show that $\mathbf{U}$ is a proper random variable if and only if

$$\widehat{K_{\mathbf{U}}} = 2K_{\hat{\mathbf{U}}}.$$

Problem 7
A bandpass signal $x(t)$ may be written as $x(t) = \sqrt{2}\Re\{x_E(t)e^{j2\pi f_0 t}\}$, where $x_E(t)$ is the baseband equivalent of $x(t)$.

1. Show that a signal $x(t)$ can also be written as $x(t) = a(t)\cos[2\pi f_0 t + \theta(t)]$ and describe $a(t)$ and $\theta(t)$ in terms of $x_E(t)$. Interpret this result.

2. Show that the signal $x(t)$ can also be written as $x(t) = x_{EI}(t)\cos 2\pi f_0 t - x_{EQ}(t)\ \sin(2\pi f_0 t)$, and describe $x_{EI}(t)$ and $x_{EQ}(t)$ in terms of $x_E(t)$. (This shows how you can obtain $x(t)$ without doing complex-valued operations.)

3. Find the baseband equivalent of the signal $x(t) = A(t)\cos(2\pi f_0 t + \varphi)$, where $A(t)$ is a real-valued lowpass signal. Hint: You may find it easier to *guess* an answer and verify that it is correct.