FINAL

Wednesday February 16, 2005, 14:15-17:15
This exam has 4 problems and 100 points in total.

Instructions

- You are allowed to use 2 sheets of paper for reference. No mobile phones or calculators are allowed in the exam.
- You can attempt the problems in any order as long as it is clear which problem is being attempted and which solution to the problem you want us to grade.
- If you are stuck in any part of a problem do not dwell on it, try to move on and attempt it later.
- The Q-function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-u^2/2} du,$$
$$\text{and} \quad Q(-x) = 1 - Q(x).$$

- $Q(x) \leq e^{-\frac{x^2}{2}}$.
- For a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

- $\sum_{n=0}^{N-1} r^n = \frac{1-r^N}{1-r}$.
- You may find this property useful. If $h \sim \mathbf{C}\mathcal{N}(0,1)$,

$$\lim_{\delta \to 0} \left[\frac{1}{\delta} \text{Prob}(|h|^2 < \delta) \right] = 1.$$

  This means that for *small* δ, $\text{Prob}(|h|^2 < \delta) \approx \delta$.
- You may find this lemma useful: If $U \sim \mathbf{C}\mathcal{N}(0, \mathbf{K}_U)$, and $\mathbf{A} = \mathbf{A}^*$, for some $b \in \mathbf{C}$ then

$$\mathbf{E}_U \left[e^{-b\mathbf{U}^*\mathbf{A}\mathbf{U}} \right] = \frac{1}{|\mathbf{I} + b\mathbf{K}_\mathbf{U}\mathbf{A}|}.$$

- There are also 5 bonus points to be had making the maximum number of points 105/100.

Good Luck!

Problem 1

[Noise cancellation **(15pts)**]
Consider two individual scalar observations Y_1, Y_2 as,

$$\begin{aligned} Y_1 &= X + Z_1 \\ Y_2 &= Z_2, \end{aligned} \qquad (1)$$

Assume that zero mean Z_1, Z_2 are independent of X and are correlated with covariance,

$$\mathbb{E}[\mathbf{Z}\mathbf{Z}^*] = \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho^*\sigma^2 & \sigma^2 \end{bmatrix}, \qquad (2)$$

where $\mathbf{Z} = [Z_1, Z_2]^T$. Let $\mathbb{E}[|X|^2] = \mathcal{E}_x$.

[*5pts*] **(a)** Find the best linear MMSE estimate of $\hat{X}$ of the random variable X from the observations Y_1, Y_2.

[*5pts*] **(b)** What is the minimum mean-squared error $\mathbb{E}[|X - \hat{X}|^2]$ of the best linear estimator? Is there a value of ρ for which we get $\mathbb{E}[|X-\hat{X}|^2] = 0$? Interpret the result if $\mathbb{E}[|X-\hat{X}|^2] = 0$ is possible.

[*5pts*] **(c)** Find the best estimate $\hat{Z}_1$ of the random variable Z_1 from Y_2. Consider the operation

$$\tilde{Y}_1 = Y_1 - \hat{Z}_1$$

Find the best linear estimate of X from $\tilde{Y}_1$. Is it the same as the answer you found in (a)? Do you have an interpretation?

Problem 2

[Inter-carrier Interference in OFDM **(25pts)**]
Consider the scalar discrete-time inter symbol interference channel considered in the class,

$$y_k = \sum_{n=0}^{\nu} p_n x_{k-n} + z_k, \quad k = 0, \ldots, N-1, \qquad (3)$$

where $z_k \sim \mathbf{C}\mathcal{N}(0, \sigma_z^2)$ and is i.i.d., independent of $\{x_k\}$. Let us employ a cyclic prefix as done in OFDM, *i.e.,*

$$x_{-l} = x_{N-1-l}, \quad l = 0, \ldots, \nu.$$

As done in class given the cyclic prefix,

$$\mathbf{y} = \begin{bmatrix} y_{N-1} \\ \vdots \\ y_0 \end{bmatrix} = \underbrace{\begin{bmatrix} p_0 & \ldots & \ldots & p_\nu & 0 & \ldots & 0 & 0 \\ 0 & p_0 & \ldots & p_{\nu-1} & p_\nu & 0 & \ldots & 0 \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \\ 0 & \ldots & \ldots & 0 & p_0 & \ldots & \ldots & p_\nu \\ p_\nu & 0 & \ldots & 0 & 0 & p_0 & \ldots & p_{\nu-1} \\ & \ddots & \ddots & \ddots & \ddots & \ddots & & \\ p_1 & \ldots & p_\nu & 0 & \ldots & 0 & 0 & p_0 \end{bmatrix}}_{\mathbf{P}} \underbrace{\begin{bmatrix} x_{N-1} \\ \vdots \\ x_0 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} z_{N-1} \\ \vdots \\ z_0 \end{bmatrix}}_{\mathbf{z}}. \qquad (4)$$

In the derivation of OFDM we used the property that

$$\mathbf{P} = \mathbf{F}^*\mathbf{D}\mathbf{F}, \tag{5}$$

where

$$\mathbf{F}_{p,q} = \frac{1}{\sqrt{N}} \exp\left(-j\frac{2\pi}{N}(p-1)(q-1)\right)$$

and $\mathbf{D}$ is the diagonal matrix with

$$\mathbf{D}_{l,l} = d_l = \sum_{n=0}^{\nu} p_n e^{-j\frac{2\pi}{N}nl}.$$

Using this we obtained

$$\mathbf{Y} = \mathbf{F}\mathbf{y} = \mathbf{D}\mathbf{X} + \mathbf{Z},$$

where $\mathbf{X} = \mathbf{F}\mathbf{x}$, $\mathbf{Z} = \mathbf{F}\mathbf{z}$. This yields the parallel channel result

$$\mathbf{Y}_l = d_l\mathbf{X}_l + \mathbf{Z}_l. \tag{6}$$

If the carrier synchronization is not accurate, then (3) gets modified as

$$y(k) = \sum_{n=0}^{\nu} e^{j2\pi f_0 k} p_n x_{k-n} + z_k, \quad k = 0, \dots, N-1 \tag{7}$$

where f_0 is the carrier frequency offset. If we still use the cyclic prefix for transmission, then (4) gets modified as

$$\underbrace{\begin{bmatrix} y(N-1) \\ \cdot \\ \cdot \\ y(0) \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} p_0 e^{j2\pi f_0(N-1)} & \dots & p_\nu e^{j2\pi f_0(N-1)} & 0 & \dots & 0 & 0 \\ \ddots & \ddots & \ddots & \ddots & \ddots & & \\ 0 & \ddots & \ddots & \ddots & e^{j2\pi f_0 \nu} p_0 & \dots & e^{j2\pi f_0 \nu} p_\nu \\ \ddots & \ddots & \ddots & \ddots & \ddots & & \\ e^{j2\pi f_0 0} p_1 & \dots & e^{j2\pi f_0 0} p_\nu & 0 & \dots & 0 & e^{j2\pi f_0 0} p_0 \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} x_{N-1} \\ \vdots \\ x_0 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} z_{N-1} \\ \vdots \\ z_0 \end{bmatrix}}_{\mathbf{z}}. \tag{8}$$

i.e.,

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z}$$

Note that

$$\mathbf{H} = \mathbf{S}\mathbf{P},$$

where $\mathbf{S}$ is a diagonal matrix with $\mathbf{S}_{l,l} = e^{j2\pi f_0(N-l)}$ and $\mathbf{P}$ is defined as in (4).

[*6pts*] **(a)** Show that for $\mathbf{Y} = \mathbf{F}\mathbf{y}$, $\mathbf{X} = \mathbf{F}\mathbf{x}$,

$$\mathbf{Y} = \mathbf{G}\mathbf{X} + \mathbf{Z} \tag{9}$$

and prove that

$$\mathbf{G} = \mathbf{F}\mathbf{S}\mathbf{F}^*\mathbf{D}.$$

[*6pts*] **(b)** If $f_0 \neq 0$, we see from part (a) that $\mathbf{G}$ is no longer a diagonal matrix and therefore we do not obtain the parallel channel result of (6). We get inter-carrier interference (ICI), *i.e.*, we have

$$\mathbf{Y}_l = \mathbf{G}_{l,l}\mathbf{X}_l + \underbrace{\sum_{q \neq l} \mathbf{G}(l,q)\mathbf{X}_q + \mathbf{Z}_l}_{\text{ICI + noise}}, \quad l = 0, \dots, N-1,$$

which shows that the other carriers interfere with $\mathbf{X}_l$. Compute the SINR (signal-to-interference plus noise ratio). Assume $\{\mathbf{X}_l\}$ are i.i.d, with $\mathbb{E}|\mathbf{X}_l|^2 = \mathcal{E}_x$. You can compute the SINR for the particular l and leave the expression in terms of $\{G(l,q)\}$.

[*6pts*] **(c)** Find the filter $\mathbf{W}_l$, such that the MMSE criterion is fulfilled,

$$\min_{\mathbf{W}_l} \mathbb{E}|\mathbf{W}_l^* \mathbf{Y} - \mathbf{X}_l|^2.$$

You can again assume that $\{\mathbf{X}_l\}$ are i.i.d with $\mathbb{E}|\mathbf{X}_l|^2 = \mathcal{E}_x$ and that the receiver knows $\mathbf{G}$. You can now state the answer in terms of $\mathbf{G}$.

[*7pts*] **(d)** Find an expression for $\mathbf{G}_{l,q}$ in terms of $f_0, N, \{d_l\}$. Given this and (b), what can you conclude about the value of f_0. For what values of f_0 do you think that the inter-carrier interference problem is important?

Hint: Use the summation of the geometric series hint given in first page.

Problem 3

[Multiple Access Channel and Transmit Code **(30pts)**]
Consider a two user multiple access channel where each user has two transmit antennas and the receiver has two antennas (see Figure).

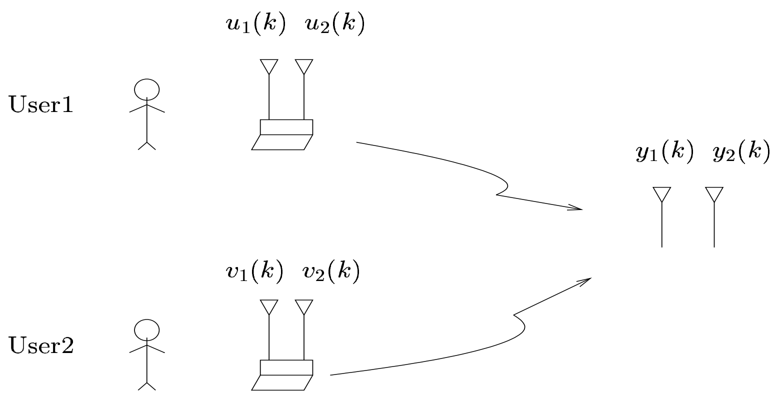

Figure 1: Multiple access channel for problem 4.

Let $\mathbf{u}(k) = \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix}$, $\mathbf{v}(k) = \begin{bmatrix} v_1(k) \\ v_2(k) \end{bmatrix}$ be the transmit signal from each user. The receiver gets a linear combination of the transmit signals,

$$y_1(k) = e_1 u_1(k) + e_2 u_2(k) + f_1 v_1(k) + f_2 v_2(k) + z_1(k),$$

$$y_2(k) = h_1 u_1(k) + h_2 u_2(k) + g_1 v_1(k) + g_2 v_2(k) + z_2(k),$$

where $z_i(k) \sim \mathbb{C}\eta(0, \sigma_z^2)$ and $\{z_i(k)\}$ is i.i.d circularly symmetric Gaussian random variables. Now, suppose the users use an Alamouti code, *i.e.*, for user 1 the transmit signal is

$$u_1(k) = a_1, \quad u_2(k) = a_2,$$

$$u_1(k+1) = -a_2^*, \quad u_2(k+1) = a_1^*,$$

and for user 2, it is

$$v_1(k) = b_1, \quad v_2(k) = b_2,$$

$$v_1(k+1) = -b_2^*, \quad v_2(k+1) = b_1^*.$$

Therefore we get

$$\begin{bmatrix} y_1(k) & y_1(k+1) \end{bmatrix} = \begin{bmatrix} e_1 & e_2 \end{bmatrix} \begin{bmatrix} a_1 & -a_2^* \\ a_2 & a_1^* \end{bmatrix} + \begin{bmatrix} f_1 & f_2 \end{bmatrix} \begin{bmatrix} b_1 & -b_2^* \\ b_2 & b_1^* \end{bmatrix} + \begin{bmatrix} z_1(k) & z_1(k+1) \end{bmatrix} \quad (10)$$

$$\begin{bmatrix} y_2(k) & y_2(k+1) \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} a_1 & -a_2^* \\ a_2 & a_1^* \end{bmatrix} + \begin{bmatrix} g_1 & g_2 \end{bmatrix} \begin{bmatrix} b_1 & -b_2^* \\ b_2 & b_1^* \end{bmatrix} + \begin{bmatrix} z_2(k) & z_2(k+1) \end{bmatrix} \quad (11)$$

[*8pts*] **(a)** Prove that (10) can be equivalently rewritten as

$$\tilde{\mathbf{Y}}_\mathbf{1} = \begin{bmatrix} \tilde{y}_1(k) & \tilde{y}_1(k+1) \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \end{bmatrix} \begin{bmatrix} e_1 & -e_2^* \\ e_2 & e_1^* \end{bmatrix} + \begin{bmatrix} b_1 & b_2 \end{bmatrix} \begin{bmatrix} f_1 & -f_2^* \\ f_2 & f_1^* \end{bmatrix} + \begin{bmatrix} \tilde{z}_1(k) & \tilde{z}_1(k+1) \end{bmatrix}$$

and (11) can be rewritten as

$$\tilde{\mathbf{Y}}_\mathbf{2} = \begin{bmatrix} \tilde{y}_2(k) & \tilde{y}_2(k+1) \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \end{bmatrix} \begin{bmatrix} h_1 & -h_2^* \\ h_2 & h_1^* \end{bmatrix} + \begin{bmatrix} b_1 & b_2 \end{bmatrix} \begin{bmatrix} g_1 & -g_2^* \\ g_2 & g_1^* \end{bmatrix} + \begin{bmatrix} \tilde{z}_2(k) & \tilde{z}_2(k+1) \end{bmatrix},$$

where $\tilde{z}_1(k), \tilde{z}_1(k+1), \tilde{z}_2(k), \tilde{z}_2(k+1)$ are i.i.d circularly symmetric complex Gaussian random variables with distribution $\mathbb{C}\eta(0, \sigma_z^2)$. Explicitly write out how $\begin{bmatrix} \tilde{y}_1(k) & \tilde{y}_1(k+1) \end{bmatrix}$, $\begin{bmatrix} \tilde{y}_2(k) & \tilde{y}_2(k+1) \end{bmatrix}$ are related to $\begin{bmatrix} y_1(k) & y_1(k+1) \end{bmatrix}$, $\begin{bmatrix} y_2(k) & y_2(k+1) \end{bmatrix}$ respectively.

[*8pts*] **(b)** Let

$$\mathbf{E} = \begin{bmatrix} e_1 & -e_2^* \\ e_2 & e_1^* \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} f_1 & -f_2^* \\ f_2 & f_1^* \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} g_1 & -g_2^* \\ g_2 & g_1^* \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} h_1 & -h_2^* \\ h_2 & h_1^* \end{bmatrix},$$

$$\mathbf{a} = \begin{bmatrix} a_1 & a_2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 & b_2 \end{bmatrix}, \quad \tilde{\mathbf{Z}}_\mathbf{1} = \begin{bmatrix} \tilde{z}_1(k) & \tilde{z}_1(k+1) \end{bmatrix}, \quad \tilde{\mathbf{Z}}_\mathbf{2} = \begin{bmatrix} \tilde{z}_2(k) & \tilde{z}_2(k+1) \end{bmatrix}.$$

Then the equations (10,11) can be written as

$$\begin{bmatrix} \tilde{\mathbf{Y}}_\mathbf{1} & \tilde{\mathbf{Y}}_\mathbf{2} \end{bmatrix} = \begin{bmatrix} \mathbf{a} & \mathbf{b} \end{bmatrix} \begin{bmatrix} \mathbf{E} & \mathbf{H} \\ \mathbf{F} & \mathbf{G} \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{Z}}_\mathbf{1} & \tilde{\mathbf{Z}}_\mathbf{2} \end{bmatrix}.$$

Prove that

$$\mathbf{W} = \begin{bmatrix} \mathbf{I_2} & -\mathbf{E}^{-1}\mathbf{H} \\ -\mathbf{G}^{-1}\mathbf{F} & \mathbf{I_2} \end{bmatrix}$$

decouples the signals from user 1 and 2, *i.e.*,

$$\begin{bmatrix} \check{\mathbf{Y}}_\mathbf{1} & \check{\mathbf{Y}}_\mathbf{2} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{Y}}_\mathbf{1} & \tilde{\mathbf{Y}}_\mathbf{2} \end{bmatrix} \mathbf{W} = \begin{bmatrix} \mathbf{a} & \mathbf{b} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{H}} & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{G}} \end{bmatrix} + \begin{bmatrix} \check{\mathbf{Z}}_\mathbf{1} & \check{\mathbf{Z}}_\mathbf{2} \end{bmatrix}, \quad (12)$$

where $\check{\mathbf{Z}}_\mathbf{1}, \check{\mathbf{Z}}_\mathbf{2}$ are still Gaussian.

[*9pts*] **(c)** Prove that $\tilde{\mathbf{H}}$ and $\tilde{\mathbf{G}}$ are of the form

$$\tilde{\mathbf{H}} = \begin{bmatrix} \tilde{h}_1 & -\tilde{h}_2^* \\ \tilde{h}_2 & \tilde{h}_1^* \end{bmatrix}, \tilde{\mathbf{G}} = \begin{bmatrix} \tilde{g}_1 & -\tilde{g}_2^* \\ \tilde{g}_2 & \tilde{g}_1^* \end{bmatrix}.$$

Hint: You do not need to explicitly write out the expressions for $\tilde{h}_1, \tilde{h}_2, \tilde{g}_1, \tilde{g}_2$.

[*5pts*] **(d)** In (12) it is seen that

$$\breve{\mathbf{Y}}_1 = \mathbf{a}\tilde{\mathbf{H}} + \breve{\mathbf{Z}}_1,$$
$$\breve{\mathbf{Y}}_2 = \mathbf{b}\tilde{\mathbf{G}} + \breve{\mathbf{Z}}_2.$$

Show that

$$\breve{\mathbf{Y}}_1\tilde{\mathbf{H}}^* = ||\tilde{\mathbf{h}}||^2 \begin{bmatrix} a_1 & a_2 \end{bmatrix} + \breve{\mathbf{Z}}_1\tilde{\mathbf{H}}^*,$$
$$\breve{\mathbf{Y}}_2\tilde{\mathbf{G}}^* = ||\tilde{\mathbf{g}}||^2 \begin{bmatrix} b_1 & b_2 \end{bmatrix} + \breve{\mathbf{Z}}_2\tilde{\mathbf{G}}^*,$$

where $\tilde{\mathbf{h}} = \begin{bmatrix} \tilde{h}_1 & \tilde{h}_2 \end{bmatrix}$, $\tilde{\mathbf{g}} = \begin{bmatrix} \tilde{g}_1 & \tilde{g}_2 \end{bmatrix}$. This completes the decoupling of the individual streams of the multiple access channel.

[*Bonus 5pts*] **(e)** If $h_1, h_2, g_1, g_2, e_1, e_2, f_1, f_2$ are i.i.d and have distribution $\mathbb{C}\eta(0, 1)$, can you guess the diversity order for detecting a_1, a_2, b_1, b_2?

Problem 4

[Relay Diversity **(30pts)**]

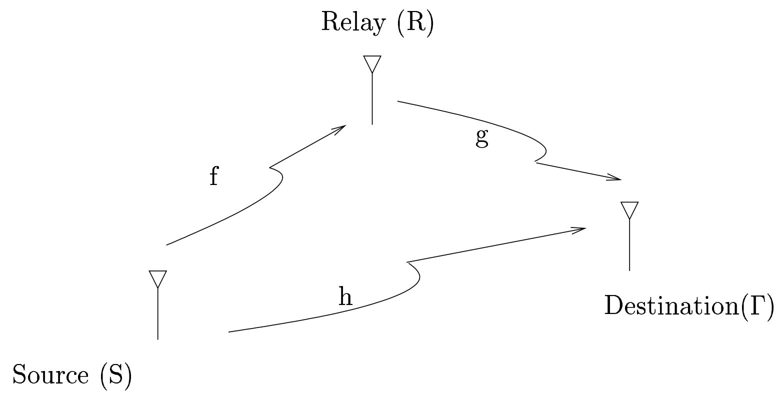

Figure 2: Communication using relay.

In a wireless network let us assume that there are three nodes, where the source ($\mathcal{S}$) wants to transmit information to the destination ($\mathcal{T}$) and can obtain help from a relay ($\mathcal{R}$). Assume that the channels are block time-invariant over a transmission block of size T. We use the following transmission protocol over a block of time of length T. Let $\{s(k)\} \in \{a, -a\}$ be a

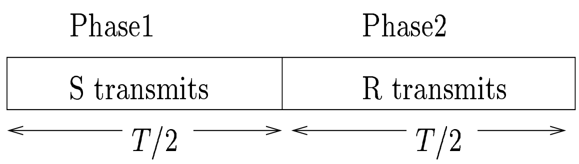

Figure 3: Transmission protocol.

binary information sequence that source $\mathcal{S}$ wants to convey to the destination $\mathcal{T}$. Then, for the first phase the relay receives $y_{\mathcal{R}}(k)$ and the destination receives $y_{\mathcal{T}}^{(1)}(k)$, with

$$y_{\mathcal{R}}(k) = fs(k) + z_{\mathcal{R}}(k), \quad (13)$$
$$y_{\mathcal{T}}^{(1)}(k) = hs(k) + z_{\mathcal{T}}(k), \quad (14)$$

where $z_{\mathcal{R}}(k), z_{\mathcal{T}}(k) \sim \mathbf{C}\eta(0, \sigma^2)$ are i.i.d, circularly symmetric complex Gaussian noise. We also assume a fading channel, $f, g, h \sim \mathbf{C}\eta(0, 1)$ and are independent of each other. Assume that g, h are known at $\mathcal{T}$, f is known at $\mathcal{R}$, but they are unknown at $\mathcal{S}$ (except for part (c)) In the second phase for parts $(a), (b)$,

$$y_{\mathcal{T}}^{(2)}(k) = gu(k) + z_{\mathcal{T}}(k)$$

where $u(k)$ is the signal transmitted by the relay and $g \sim \mathbf{C}\eta(0, 1)$.

[*6pts*] **(a)** Suppose the relay was absent, *i.e.,* $u(k) = 0$, then give an expression (or bound) on the average error probability of $\{s(k)\}$ averaged over the channel realizations h. What is the diversity order, *i.e.,*

$$\lim_{\text{SNR}\to\infty} \frac{-\log(\bar{P}_e(\text{SNR}))}{\log(\text{SNR})}$$

for the detection of $\{s(k)\}$.

[*12pts*] **(b)** Suppose that the relay $\mathcal{R}$ attempts to decode $\{s(k)\}$ in phase 1 and transmits $u(k) = \hat{s}_{\mathcal{R}}(k)$ in phase 2. That is, it sends the decoded sequence to $\mathcal{T}$. Assume now that there is an oracle which tells the destination $\mathcal{T}$ if relay $\mathcal{R}$ has decoded correctly or not. Note that the oracle just lets $\mathcal{T}$ know if $\hat{s}_{\mathcal{R}}(k) = s(k)$ but *not* its value. Now, $\mathcal{T}$ can use the received sequence from both phase 1 and phase 2 in order to decode $\{s(k)\}$. Find expressions (or bounds) for the error probability for this decoder averaged over the channel realizations which achieve the *best* diversity order at $\mathcal{T}$. What is the *best* diversity order that can be achieved at $\mathcal{T}$ for detecting $\{s(k)\}$?

Hint: Develop a receiver strategy that uses the information given by the oracle. You do not need very detailed calculations for obtaining error probability bounds.

[*12pts*] **(c)** Suppose now that we have a new protocol. Phase 1 is as before, where $\mathcal{S}$ transmits and both $\mathcal{R}$ and $\mathcal{T}$ receive the signal as in (13) and (14) respectively. At the end of phase 1, there is a feedback channel from $\mathcal{R}$ to $\mathcal{S}$ which informs $\mathcal{S}$ about the realization of channel f. Now the protocol $\mathcal{S}$ and $\mathcal{R}$ follow is given by: if $|f|^2 \le c(\text{SNR})$ (where $c(\text{SNR})$ is a function of SNR), then in phase 2, $\mathcal{S}$ repeats the same information it transmitted in phase 1 and the relay $\mathcal{R}$ remains silent, *i.e.,* $\{s(k)\}$ from phase 1 is repeated and $u(k) = 0$ in phase 2. If $|f|^2 > c(\text{SNR})$, then the protocol is as in part (b), *i.e.,* $\mathcal{S}$ remains silent and $\mathcal{R}$ sends the decoded information $u(k) = \hat{s}_{\mathcal{R}}(k)$. Let $c(\text{SNR}) = \frac{1}{\text{SNR}^{1-\epsilon}}$ for an arbitrarily small $\epsilon > 0$,

Assume that an oracle informs the receiver $\mathcal{T}$ whether in phase 2, $\mathcal{S}$ or $\mathcal{R}$ is transmitting. If in phase 2, $\mathcal{S}$ is transmitting, the receiver $\mathcal{T}$ forms the decision variable,

$$\tilde{y}_{\mathcal{T}} = \begin{bmatrix} h^* & h^* \end{bmatrix} \begin{bmatrix} y_{\mathcal{T}}^{(1)} \\ y_{\mathcal{T}}^{(2)} \end{bmatrix}$$

where $y_{\mathcal{T}}^{(1)}, y_{\mathcal{T}}^{(2)}$ are the received signals in phase 1 and phase 2 respectively. On the other hand if in phase 2, the relay $\mathcal{R}$ is transmitting, receiver $\mathcal{T}$ forms the decision variable

$$\tilde{y}_{\mathcal{T}} = \begin{bmatrix} h^* & g^* \end{bmatrix} \begin{bmatrix} y_{\mathcal{T}}^{(1)} \\ y_{\mathcal{T}}^{(2)} \end{bmatrix}$$

where again $y_{\mathcal{T}}^{(1)}, y_{\mathcal{T}}^{(2)}$ are the received signals in phase 1 and phase 2 respectively. The decision rule in both situations is that $\mathcal{T}$ chooses $\hat{s}_{\mathcal{T}} = +a$ if $\Re(\tilde{y}_{\mathcal{T}}) \ge 0$ and $\hat{s}_{\mathcal{T}} = -a$

otherwise. Analyze the performance of this receive strategy *i.e.,* find the diversity order that can be achieved by $\mathcal{T}$ for $\{s(k)\}$. Note that we are looking for diversity order and so we do not necessarily need a detailed analysis to find the diversity order.

Hint: Condition on appropriate error events at the relay and use error probability bounds. You can also use properties given in the hints in the first page.