A generalization of Cauchy’s determinant identity

Let $H_k$ be the $n \times n$ matrix defined as

$$h_{ij} = \frac{1}{(x_i + y_j)^k}$$

where $x_i, y_j$ are arbitrary positive numbers and $k$ is a positive integer. Cauchy’s determinant identity reads

$$\det(H_1) = \frac{\Delta(x) \Delta(y)}{\prod_{i<j}(x_i + y_j)}$$

where $\Delta(x) = \prod_{i<j}(x_j - x_i)$.

Borchardt then showed that

$$\det(H_2) = \det(H_1) \operatorname{perm}(H_1)$$

where $\operatorname{perm}(H_1)$ is the permanent of the matrix $H_1$. This allows to conclude that $\det(H_2)$ is of the form

$$\det(H_2) = \frac{\Delta(x) \Delta(y) P_2(x, y)}{\prod_{i,j=1}^n(x_i + y_j)^2}$$

where $P_2(x, y)$ is some polynomial with non-negative coefficients.

The conjecture is: for any integer $k \geq 1$, we have

$$\det(H_k) = \frac{\Delta(x) \Delta(y) P_k(x, y)}{\prod_{i,j=1}^n(x_i + y_j)^k}$$

where $P_k(x, y)$ is again some polynomial with non-negative coefficients.

This conjecture was made by Emmanuel Preissmann.